High School Teacher and Her Students' Use of Representations while Using the CAS-Intensive Mathematics Curriculum

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Introduction

Secondary mathematics teachers undertake the perennial tasks of planning and orchestrating learning environments for their students. Among the many decisions teachers make are selecting and guiding appropriate mathematical activities and incorporating suitable technology tools. Students then pursue and transform these activities into their own experiences. This paper reports about one teacher and her class while exploring two general questions: What is the nature of students' use of representation in the technology-intensive environment of this classroom? What relationships emerge among the mathematical tasks, uses of technology and mathematical representations encountered and generated by these students?

Literature Review

Teachers struggle with the time and effort needed to incorporate technology within the existing curriculum (Lampert, 1993, Wiske & Houde, 1993). Lampert observed teachers experiencing a liminal period of transition from their usual classroom experiences toward something new; i.e., times when established norms were challenged within a technology-enhanced inductive inquiry approach to learning geometry. The tension between open-ended inquiry and structured school work can result in barriers for both teachers and students (Yerushalmy, Chazan, & Gordon, 1993). Some of these obstacles include doubt, confusion, dead ends, frustrations, and assessments that do not match the inquiry tasks. For the teacher, tensions persist in trying to orchestrate a balance between open-ended inquiry and structured schoolwork. This balance may be particularly elusive for teachers with procedural views of mathematics as they tend to prefer and impose highly5structured classroom events while their students are using technology to explore freely mathematical ideas (Tharp, Fitzimmons, & Ayers, 1997; Zbiek, 1995, In press).

Technology use is one of several factors that may influence teaching and learning in the mathematics classroom. A teacher's view of technology as curriculum rather than as a tool for learning mathematics manifests itself in the classroom with teacher-specific student directions on how to use the technology (Heid, Blume, Zbiek, & Edwards, 1999). Teachers who have a rule-based concept of mathematics tend to use a highly structured approach when teaching with technology (Turner & Chauvet, 1995).

Teachers are key figures in what occurs in the classroom. They provide the climate and structure for the presentation and development of mathematical ideas during classroom instruction. In the interest of understanding more clearly students' use of mathematical representation in a technology-intensive environment and the relationships among the mathematical tasks, uses of technology and mathematical representation, I studied one teacher's implementation of the technology-intensive mathematics curriculum.

Methodology

Setting

The 85-minute block-scheduled class consisted of 31 students enrolled in one heterogeneous section of algebra II course in a medium-size high school in a suburban Midwest area. There were five grade-10, nineteen grade-11, and seven grade-12 students in the class. Seven of the thirty-one students were classified as special education, so in addition to the regular teacher there was a collaborator teacher. Upon entering this experimental course the typical student had taken algebra IA and algebra IB (Each of these counts individually as one-half mathematics credit toward graduation) as well as geometry. There is a 3-year mathematics requirement for graduation; so the current experimental course likely will be the last high school mathematics course for many of these students.

The class was equipped with 14 IBM/clone model 486-computers around the perimeter of the back half of the room, and six round tables in the front half of the room with four to six students at each table. The students had immediate access to *The Geometer's Sketchpad* (Jackiw, 1995) on the computers and each student had access to a TI–89 calculator in conjunction with the CAS-Intensive Mathematics (CAS-IM) Project curriculum in workbook form. The workbooks and some accompanying computer files were generated by the CAS-IM Project. The project was funded by the National Science Foundation to create and field test curriculum modules to implement mathematics technology in secondary school mathematics courses beyond introductory algebra.[1]

Subject

Kristin was in her fifth year of teaching secondary mathematics. She has a B.A. in mathematics and a M.A.T. in secondary mathematics education. None of her undergraduate mathematics courses made use of technology. While pursuing her graduate degree, Kristin used the SAS statistical package in a statistical data analysis course and a computer algebra system (either Maple or Mathematica) in a

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mathematics course on applications for secondary education. The technology focus in another graduate mathematics education course was using graphing calculators in 7-12 mathematics classrooms. Kristin had some experience incorporating technology into her teaching of mathematics. She had used the dynamic geometry feature of the TI-92 while teaching geometry courses, including one course using geometry-focused modules of the CAS-IM curriculum materials during the previous semester. However, this was her first experience using a computer-algebra system while teaching algebra.

Analysis

The researcher videotaped Kristin's algebra II class on six different occasions during February, March, and April (three sets of two-consecutive days) during the spring semester. These tapes were transcribed verbatim and were annotated using the videotape, field notes and notes taken by the researcher during pre/post interviews with the teacher. These annotated transcripts were coded using technology-use codes (Zbiek, 2002b), MAGICAL framework codes (Zbiek, 2002a), and task-type codes (Heid, Blume, Hollebrands, & Piez, 2002).

This paper reports on one set of the videotaped classes. As a beginning point in the analysis of the data, I chose the two April days because of the richness of the classroom interactions. There was a mix of computer lab activity using a *Geometer's Sketchpad* (Jackiw, 1995) dynamap, parameter exploration using the TI-89, and whole-class instruction. The mathematical content included ceiling function, floor function, and logarithmic function and its inverse.

Results and Discussion

The discussion begins by examining the representation actions that emerge during the two classes with a closer look at the dominant representation types. This leads into a discussion of the role of technology use and of task type use in the type of representation action.

Representation Uses in the Context of Tasks Types

Figure 1 displays the six representation types that emerged from the coding. The ordered-triple coding describes the type of representation – same or different (e.g., graphical, symbolic, numerical), the mathematical thing represented – same or different (e.g., function, ordered pair), and the nature of the use of the representation (i.e., the MAGICAL categories). I coded the actions of the teacher, her students, and the collaborator teacher in an effort to describe their uses of the shared representations. Student representation use occurred in concert with the teacher and occasionally with the co-teacher. For the 51 representation-use codes, there were 24 such occurrences, in addition to 25 representation uses central to Kristin and two uses central to the collaborator teacher.

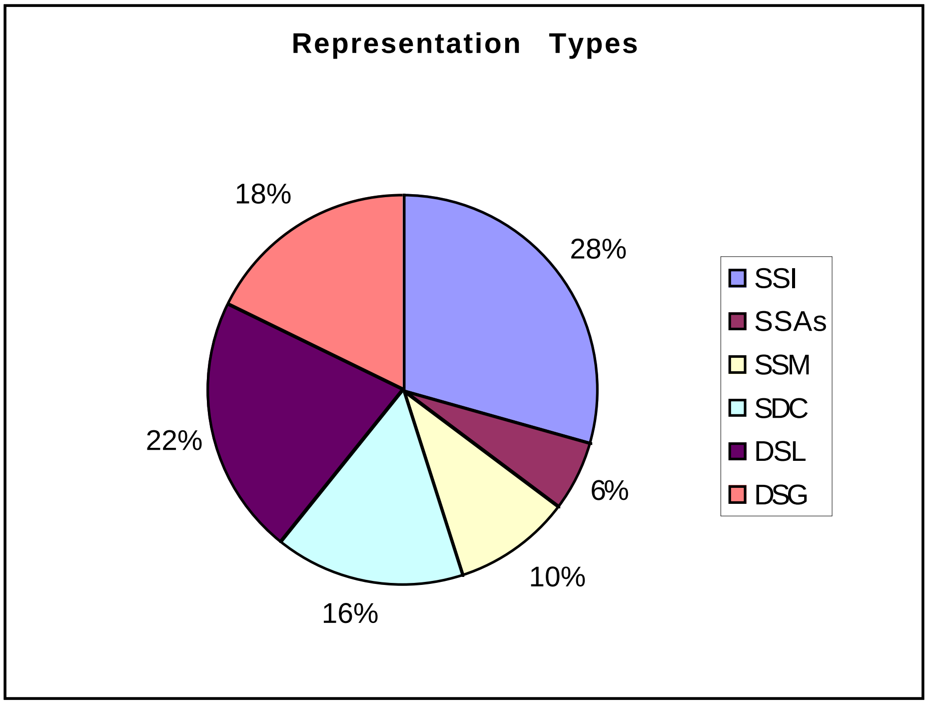

Figure 1. Representations as percent of 51 occurrences

The dominant representation use is Interpretation (SSI) – 15 out of 51 times. This many may not be surprising in a classroom using the fourth CAS-IM curriculum module (Zbiek, In progress/In field test). The curriculum presents mathematical ideas through situations that are intended to be familiar to the students. The class then spends a significant amount of time Interpreting representations as they discuss how the mathematical aspects of the representations relate to the situations. Eleven of the 51 occurrences involve Linking a representation of one type to a representation of another type (DSL). Nine of the occurrences consist of Generate a first representation of a new type from an existing representation of another type (DSG). Connecting two different mathematical things within the same representation type (SDC) appear in eight of 51 occurrences.

Each representation type arose in a variety of task types. For example, Interpretation (SSI) was used during Produce Value (PE), Describe Observation

(DO), Describe Procedure (DP), and Compare/Elaborate/Describe phenomenon (CED) tasks. Describe Procedure (DP) and Generate Function Specifics (GFS) tasks seem to lead to Generating (DSG) actions. Compare/Elaborate/Describe phenomenon (CED) tasks seem to elicit Linking actions (DSL) and Connecting actions (SDC).

Task Types with Major Curriculum Episodes

An episode is a unit from the classroom transcript that describes a sequence of actions by the students and the teacher. The sequence narrates their discussion of the mathematical ideas encompassing a curriculum task. An episode transcends "task" as defined by Heid and her colleagues (2002). Rather, an episode embodies Doyle's (1988) four aspects of classroom work: "(a) a goal state or end product to be achieved; (b) a problem space or set of conditions and resources available to accomplish the task, (c) the operations involved in assembling and using resources to reach the goal state or generate the product, and (d) the importance of the [curriculum] ask in the overall work system of the class" (p. 169). An episode also includes subtasks added by the teacher or students.

Among the five large curriculum task episodes, three of them are GFS episodes, one is a CED episode, and the fifth is a Pt (Predict) episode. It is interesting to note the pattern of small subtasks emerging within the three GFS episodes. Figure 2 illustrates this pattern. All three of the GFS episodes include Produce Value (PE), Describe Procedure (DP), and Generate Function Specifics (GFS) subtasks. The PE task may occur first or directly following a DP task. The episode ends with a GFS task.

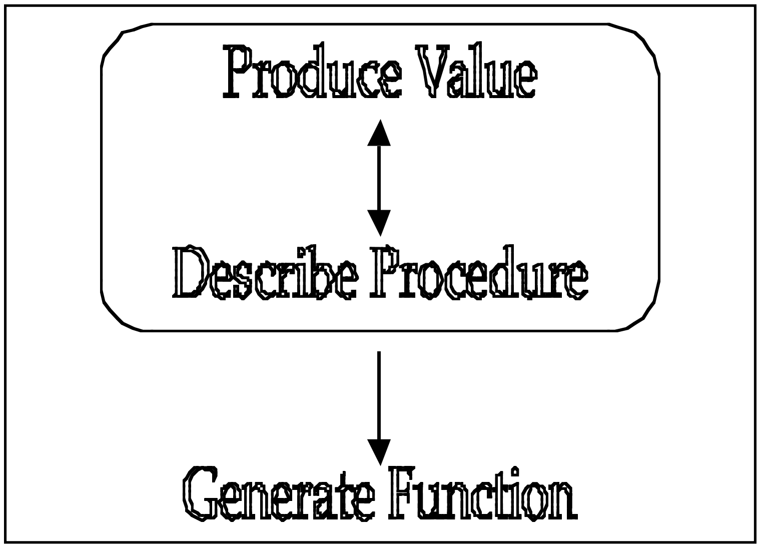

Figure 2. Subtasks' pattern within large GFS episode

As an example, consider the GFS curriculum task based on the Fabric Situation, item 2 from Module IV, Section 2, of the CAS-IM curriculum (Zbiek, In progress/In field test). The curriculum task actually begins with the problem assigned to students as homework on the previous day. The textbook material for the homework problem and the Fabric Situation appear in Figure 3.

Fabric Situation. Sally Glass often buys fabric for sewing and crafts. Her favorite fabric costs $7.95 per yard. The fabric store charges her by the eighth-yard. For example, if she buys 0.5 yards, she pays $\frac{4}{8}$ of $7.95. If she buys $\frac{1}{5}$ yard, she pays for $\frac{2}{8}$ yards since $\frac{1}{8} < \frac{1}{5} \leq \frac{2}{8}$. The cost is always rounded to the next highest cent, not to the nearest cent.

1. Complete the following tables representing the cost of Sally's fabric.

Table 1:

Number of Yards	Number of Eighths of Yards	Number of Eighths Paid for	Cost of Fabric in Dollars
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1			

2. Write a function rule that shows cost of the fabric as a function of the number of yards. You should be able to use this function rule to produce two tables similar to those shown in Exercise 1.

Figure 3. Fabric Situation, Exercise 1 Table, and Exercise 2

As the episode plays out, Kristin poses several subtasks that are not part of the written curriculum. She begins by reading item 2 (Figure 3) followed by, "Okay, what was the function rule we came up with?" She then says, "Essentially, you guys, we're just writing down what we did to get from the first box to the last one on the table. Okay, all right, so what did we have now?" I see that she initially states the Generating (GFS) task but seemingly suggests a Procedure approach as she suggests that the students focus on the actions necessary to move from column to column in the table (Figure 3). The students are using the table (Figure 3) representing the cost of Sally's fabric that they had completed as homework. This Produce Value (PE) task evokes an Ascribing (SSAs) action since it is the first representation of the Fabric Situation.

She continues interacting with the class as one student, Alan (a pseudonym), tries to describe his answer to the question. As he motions with his hands and traces the ceiling symbol in the air with his finger, Kristin writes on the board: $c(x) = \left\lceil \frac{f}{8} \right\rceil$. Struggling to capture his idea, Kristin finally has Alan come to the board to write the function rule: $c(x) = \left\lceil \frac{f}{8} \bullet 7.95 \right\rceil$. This is the first Generating (DSG) action of this episode.

The curriculum task is to generate from the table representation the symbolic representation for the Fabric Situation. Alan has generated a rule, but Kristin suggests more needs to be done as she says, "I kind of agree with him, but he's missing one thing. [*Pause*.] He's missing one thing. Let's think about … Let's look at the steps. Let's look at the steps that we did." In an effort to modify Alan's rule, Kristin breaks the task into smaller Describe Procedure (DP) tasks as she says, "Okay, number of yards. To go from the number of yards to the number of eighths of a yard, what did we do? Look at your table. What did we do to go from that first column to the second column?" After several students respond at once, Kristin writes the rule, $8x$, on the board as she says, "Multiplied by eight. Let's just work with x being the number of yards. First thing we did was multiply that by eight." This is the second Generating (DSG) action in the episode.

Kristin continues the process by asking the students to describe their procedure that results in values in the third column given the values in the second column of the table (Figure 3) as she says, "How can we get from the number of eighths of a yard to the number of eighths paid for?" Alan responds, "Ah, rounded", then Kristin Generates (DSG) the third rule, $\lceil 8x \rceil$, as she writes on the board and says, "Okay, ah, we, we did the ceiling. All right, we did the ceiling of this." She is also using a Manipulating (SSM) action as she mentally composes two rules. The rule that takes input values, *y*, from the second column of the table (Figure 3) and produces output values, $z(y) = \lceil y \rceil$, in the third column is composed with the previous rule, *y(x)*=8*x*, that takes input values from the first column and produces output values in the second column; that is, $z(y(x)) = z(8x) = \lceil 8x \rceil$.

The process continues as Kristin says, "Now, how did we get from the number of eighths paid for to the cost of the fabric in dollars? " and Alan describes his procedure (DP) for obtaining the values in the fourth column, "Divided by eight and multiplied by seven point nine five." Kristin again uses Generating and Manipulating actions to obtain the rule, $\lceil 8x \rceil \frac{7.95}{8}$, for the fourth column of the table. If we let *w(z)* represent the output values, Kristin's mental actions are $w(z(y(x))) = w(\lceil 8x \rceil) = \lceil 8x \rceil \frac{7.95}{8}$. The final Generating (DSG) action results as Kristin summarizes the three Describe Procedure (DP) subtasks and writes the rule on the board, $c(x) = \lceil 8x \rceil \frac{7.95}{8}$, while saying, "Okay. So that would be what our …[*pause*]. That would be our function rule right here. [*Kristin points to what she has written on board.*] We could write c of x equals ceiling of eight x times seven point nine five divided by eight." These three Describe Procedure (DP) tasks by Alan and last rule generation (GFS) by Kristin give the final Generating (DSG) action. This final action has the previous three Generating (DSG) actions are subcategories and changes Alan's representation for the function rule to another representation.

Figure 4 shows the progression of these subtasks and the types of representation actions that occur. The curriculum task is Generate Function Specific (GFS) and T_Tasks are those proposed by Kristin during the discussion of the Fabric Situation item with the exception of the Generating task in Italics. This GFS task represents Alan's attempt to Generate the function rule that is then modified until the actual function for the Fabric Situation is Generated.

T_Task	**Representation Use**	
PE	→ SSAs	
GFS	→ DSG	
DP	→ DSG & SSM	} DSG
DP	→ DSG & SSM	} DSG
DP	→ DSG & SSM	} DSG
GFS		} DSG

Figure 4. Subtask patterns within Fabric Situation GFS curriculum task

Technology and Representation Uses

During this episode, Generating (DSG) was the dominant MAGICAL category with Representation Generator (RG) technology use. Kristin used the board for RG technology use purposes. Through the board she conveyed the symbolic rules as they arose. This pattern of board use was consistent throughout the two-day observation. The students always had access to TI-89 technology, but it was not always clear how they were using it, except when Kristin also used the TI-89 technology for Generating and Manipulating graphical representations.

It is not surprising that Representation Generator (RG) affords the Generate (G) MAGICAL category. It is interesting to observe what other technology uses may afford representation uses. Both Information Conveyor (IC) and Example Generator

(EG) affords the Interpret (I) MAGICAL category with the use of dynamic geometry technology. I never observed Kristin using dynamic geometry technology in the classroom, so all of the occurrences result from student use during a laboratory setting. Manipulation Aide (MA) and Attention Helper (AH) technology uses also surfaced as facilitating Manipulate (M) and Link (L) MAGICAL categories, respectively.

Physical Object (PO) in conjunction with board use was the dominant technology type for linking different representation types for the same mathematical thing (DSL). Often Kristin and sometimes her students would use hand and arm motions trying to convey ideas about the object under discussion. The dominant type of technology for Kristin is board (PP) during Generating and Manipulating actions. This seems in concert with Kristin's belief about electronic technology use with students. During an interview, she said, "I would not use technology when teaching fundamental concepts or skills." Kristin's dominant board use seems to convey her belief that it is important for students to know how to do the mathematics by-hand when they are expected to master the concept or skill. On the other hand, she enjoyed and saw value in using electronic technology during her teaching, but often struggled with the knowledge that her students would need to demonstrate certain concepts and skills on the state wide mandated test in the absence of any electronic technology use.

Comparing/Elaborating/Describing phenomenon (CED) was the dominant task during the two-day observations, occurring 24 (10 curriculum, 13 teacher, and one collaborator tasks) out of a total of 81 tasks. Eight of Kristin's 13 CED tasks were modifications of the CED curriculum tasks as she added detail while clarifying.

Kristin would always read the task out loud and would often follow with a change in the wording, a clarification, or separation of the task into smaller parts. An example of the latter was discussed earlier in the Fabric Situation (Figure 3) as Kristin and her students generated the function rule for purchasing fabric. Kristin initiated all fifteen of the Describe Procedure (DP) tasks. These task types most often culminated in the occurrence of Generating (DSG) types of use of representation.

The Teacher's Perspectives

Early in the Spring Semester during an interview I asked Kristin to describe her typical class prior to using the CAS-IM curriculum. She described it as consisting of three major parts: 1) "Go over homework or do something from the previous day," 2) "discuss the new material that may include using technology," and 3) "assign practice problems." It appeared that Kristin and her students' experiences of tension between open-ended inquiry and structured schoolwork are consistent with those reported in Yerushalmy, Chazan, and Gordon (1993). The established structure of Kristin's classroom was being challenged as the experience of Lampert's (1993) liminality surfaced. Kristin and her students were at the threshold of establishing new classroom norms for learning and teaching mathematics in a technology-intensive environment.

Kristin also experienced these tensions during the Fall Semester using the CAS-IM curriculum in a geometry course. This is evidenced in an earlier interview as she spontaneously talks about her experiences using the CAS-IM curriculum: "The curriculum is complicated. You can't give it to just any teacher like I had wanted to. A lot of the curriculum is having kids figuring it out. Even Jennifer [*her* *special education collaborator*] sits here and gets frustrated trying to figure things out. Lots of kids want to understand clearly, but they can't. The curriculum always has kids figuring it out and making conjectures. For example, shears! Even I didn't know what a shear was." Shears were not part of Kristin's usual high school teaching curriculum, but she was faced with teaching this new and unusual geometric transformation in new ways. The use of a dynamic geometry tool made shears accessible to both Kristin and her students. It afforded access to a mathematical concept that traditionally was first encountered at the collegiate level. This liminal experience placed Kristin and her students at the threshold of a technology-intensive environment that facilitates explorations between and among representations in dynamic as well as static ways.

During the classroom episodes, the students' representations and technology uses are influenced by the teacher's use. Kristin demonstrated some of the same characteristics as found in the Tharp, Fitzimmons, and Ayers (1997) and Zbiek (1995) studies. She structured the inquiry activities and would lead the class step-by-step. This seemed to result from her concern that every student understands how to use the technology while working on the task. For most of her students, this was their first experience with CAS and dynamic geometry in a curriculum that makes use of inquiry activities as part of the development of the concepts.

Kristin struggles with "doing" the mathematics on paper verses using the technology while learning the mathematics. She almost approaches technology as curriculum (Heid, Blume, Zbiek, & Edwards, 1999). She tends to lean toward learning the mathematics first and then using the technology: "The problem is making the transition from computer to paper. Maybe we should do it on paper first and then go to the computer… Maybe they could be asked to do the same thing on paper as on computer." Rather than using technology as a tool for exploring mathematical ideas, Kristin seems to view technology as part of the curriculum to be learned. This approach is similar to that of the preservice teachers in the Turner and Chauvet (1995) study and to the veteran teachers described in the Heid, Blume, Zbiek, and Edwards (1999) case studies report.

Conclusion

Four different types of representation actions were prominent during the two-day classroom observation:

- (SSI) Interpret the same mathematical object within the same representation type,
- (DSL) Link the same mathematical thing within two different representation types,
- (DSG) Generate a new representation type from an existing representation type for a mathematical object, and
- (SDC) Connect different mathematical objects within the same representation type.

Examining the role that technology use and task type play out, patterns emerged for three of these four representation types.

Certain task types seem to elicit particular representation actions as described in Figure 5. Produce Value (PE) task use seems to elicit Interpreting action SSI. Describe Procedure (DP) and Generate Function Specifics (GFS) task uses seem to evoke Generating action DSG. Compare/Elaborate/Describe phenomenon (CED) task use seems to elicit Linking action DSL. Likewise, certain technology uses seem to facilitate these representation actions. Information Conveyer (IC) and Example

Generator (EG) with *Sketchpad* use seem to assist Interpret (SSI). Representation Generator (RG) with board use seems to facilitate Generate (DSG). Attention Helper (AH) via hand and arm motions seems to advance linking (DSL).

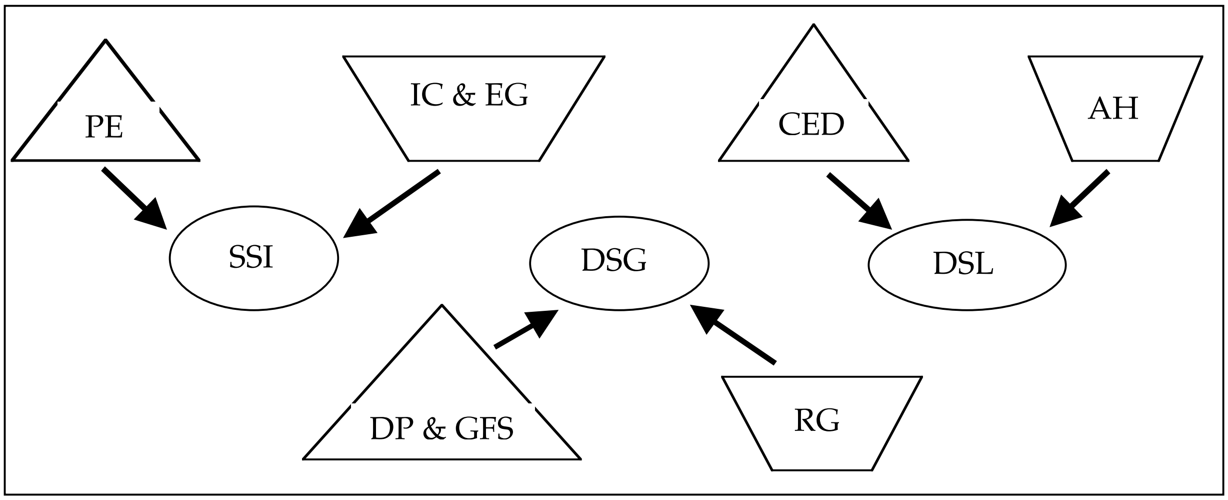

Figure 5. Relationships among representation types and task/technology uses

This gives us a glimpse about representation uses in the technology-intensive environment of this classroom. It will be interesting to see how these patterns compare with future analysis of classroom interactions using MAGICAL representation use categories coupled with the purposes in using technology for mathematical tasks.

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