# Patterns in Students' Engagement with Tasks in a Technology-Intensive Secondary School Mathematics Curriculum

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## Introduction

When mathematics teaching and learning occurs in technology-intensive environments, students often engage with tasks and use resources that differ markedly from what might be available in environments that do not make calculator and computer technology available to students. Exploration, generalization, reasoning, and justification are but a few of the processes which technology can facilitate. Because students' performance on various tasks may be influenced by the resources available to them (e.g., multiple linked representations) in a technology-intensive environment, one step in understanding that potential influence can be to determine the nature of or patterns in students' engagement with the tasks they encounter in technology-intensive environments.

This study addressed the research question: What are the patterns of students' engagement with mathematics tasks in the context of a technology-intensive curriculum? The way in which the notion of "task" was defined in this study differed somewhat from the way "task" has been defined in other research literature. Our concept of "task" involved the following assumptions:

- Tasks are goal-driven and require student action and "new" thinking. This assumes that students are not just reporting previously recorded findings.
- Tasks that to be coded are those that are posed by the researcher, curriculum, teacher, and students. Tasks may be implicit (inferred as being the task from students' actions and statements) or explicit (posed verbally or in the curriculum materials).
- While students are working on a large task, such as the one posed by the curriculum, subtasks may be introduced [tasks will overlap].

In a companion paper, "The Development of a Mathematics Task Coding Instrument," we delineate in more detail the notion of "task" and give the rationale for and a description of the development of a coding instrument that we used to code tasks.

## The Research Setting

### School and Curriculum

The school in which the study took place was a medium-size high school in a suburban Midwest area. During the Fall semester of 1999 six teachers from the school's mathematics department expressed interest in learning about new technology-intensive secondary mathematics curricula. During the Spring semester of 2000 those six teachers participated in three two-day workshops developed by two members of the research team. The workshops focused on technology-intensive mathematics curricula; in particular, they were designed to acquaint the teachers with the *Computer Algebra System-Intensive Mathematics* (CAS-IM) curriculum (Heid, Zbiek, and others, in preparation). One of the teachers offered her classroom as a site for the research team to conduct the classroom-based research studies.

The study took place in a heterogeneously grouped second-year algebra class. The course was offered in a block-scheduled format–classes that were 85 minutes in length and met five days per week for one semester. The course used CAS-IM Modules II (Function Composition and Inverse Functions), IV (Families of Functions), and IX (Symbolic Reasoning) as its primary curriculum. During class and outside of class each student had access to a computer algebra system (TI-89 calculator), and pairs of students had access, in the classroom, to computers with The Geometer's Sketchpad (Jackiw, 199x) software.

### Subjects

The class consisted of 31 students (five in grade 10, nineteen in grade 11, and seven in grade 12). Seven of the 31 students were classified as special needs students, so there was a collaborator teacher in the classroom in addition to the regular teacher. The typical student in this course had taken Algebra IA and Algebra IB (each counting as one-half mathematics credit toward graduation) as well as Geometry. The school has a three-year mathematics requirement for graduation, so this experimental second-year algebra course was anticipated to be the last high school mathematics course for many of the students in the class.

From the 31 students in the class 12 volunteers were selected as target subjects for the study. During the classes in which small-group problem solving was observed, the researchers, in conjunction with the classroom teacher, assigned students to groups of three or four that consisted of students with roughly comparable achievement in the course.

## Data Collection and Analysis

### Data Collection

Data were collected from the 12 target students' work in small groups during two two-day sequences of classes (two consecutive days in late March and mid-May). The two sets of two small-group observation days with the 12 target subjects occurred subsequent to the second and third individual interviews that were conducted with those subjects in March and April, respectively (see Figure 1). During their small-group work in class, the twelve subjects were divided into three groups.

The tasks on which students worked during the days on which small-group data were collected were composed of the CAS-IM curriculum materials that the students used at that point in the course and materials related to them. A sample of the curriculum materials from which the students worked in their small groups appears in Figure 2.

Four members of the research team were present during data collection. One of the research team members served as the primary classroom teacher on the days on which the small-group data were collected. The research team member conducted class (introduced the lesson, called students together to discuss or summarize their work in small groups, and directed students to the next task at appropriate times) and facilitated the small groups of non-target students (primarily pairs). The regular classroom teacher and collaborator also assisted and observed the non-target students during their small group work. The remaining three members of the research team served as facilitators for the target students in their small groups, asking questions, eliciting clarification of students' statements, and suggesting things for the group to explore.

A TI-Presenter and VCR captured on video tape each group's calculator (one per group) or computer use. An audio tape fed by individual lapel microphones captured students' discussion and interaction with the researcher and teacher. A video camera recorded students' written work and their interactions with their group. Copies of students' papers from class provided a record of their written work.

### Data Preparation

A transcriber produced complete verbatim transcripts of each of the small-group sessions. Members of the research team proofread and annotated the transcripts with students' gestures, calculator and computer screen dumps, and students' written work.

### Data analysis

Building on categories of tasks from the research literature, four members of the research team read transcripts initially to identify tasks. These tasks were expanded to include tasks presented by the curriculum, tasks as interpreted by the students (as inferred from what the students did to address the task), tasks presented by the teacher to the whole class, tasks presented by one of the researchers to a small group, and tasks presented by a student to other members of that person's small group. From the transcript analyses a process of code-define-recode-redefine-recode was repeated to refine the categories and codes. See "The Development of a Mathematics Task Coding Instrument" for a more complete description of the development of the task categories and codes.

This paper focuses on data collected from one of the three small groups (Hank, Jenny, Jim, and Rachel–all names are pseudonyms) during the March two-day small group observations (the first of the two two-day sessions). This group was selected because of completeness of data, extent of continued membership in the same group throughout the small group observations, and potential richness of the group's discussion.

## Patterns in Students' Engagement with Tasks

Locating curriculum tasks in the transcript and tracing the sequence of subtasks on which students worked yielded sequences of tasks that were analyzed to identify patterns in task engagement in a technology-intensive context. Five patterns of students' engagement with tasks were identified initially. These patterns were labeled as:

(1) Task-Technology-Corroborate;

(2) Task-Technology-Task*;

(3) Technology as Perturbation Generator;

(4) Partitioning and Ramping up to the Task;

(5) Reaching an Impasse.

Although these patterns were the most salient ones during the initial analysis, it is possible that subsequent analysis of the data will reveal additional patterns or modifications of these five patterns.

Task-Technology-Corroborate

The Task-Technology-Corroborate pattern was identified in various places and tended to occur when students were asked to evaluate an expression or to compare/explain/describe mathematical concepts or ideas. For example, students were asked to Define $h(x) = \lfloor x \rfloor$ and use this function rule to complete the table shown in Figure 3. This curriculum task was coded as a Produce Evaluate task because students were asked to evaluate the function rule for specific input values.

An example of the Task-Technology-Corroborate pattern occurred when Jim explained that he could also produce the output values using his head instead of the calculator. Jim stated aloud that the corresponding output values for the input values listed in the first column would be –2, -1, 0, 0, 2, 2. Rachel entered $h(-1.2)$ into the TI-92 calculator, and then she asked Jim what result he obtained for this problem. In this case Rachel used the technology to engage in the Produce Evaluate task posed by the curriculum and used technology to corroborate Jim's by-head work. This pattern of working on the original task, using the technology, and then checking with others the correctness of the procedure, results, or strategy was common.

Task-Technology-Task*

Another pattern that was identified was that of working on a task, using the technology, and creating a task for the purpose of making connections between the original task and other mathematical concepts and ideas. This pattern was coded as Task-Technology-Task*.

At times students would begin working on the curriculum task, use the technology to investigate the problem, and a new task would be posed, usually by a student, that would connect to other mathematical concepts or ideas related to the original task. For example, while students were evaluating the floor function for specific input values and using the technology to assist them in obtaining these values, Jenny noted that $floor(-1.2) = -2$. She explained that this result made sense because the floor was rounding down. She introduced a new task which was to find $ceiling(-1.2)$. She stated that because ceiling rounds up and floor rounds down, the ceiling of (- 1.2) would be – 1. The new task that was posed to the group was related to, but different from, the original curriculum task. It appeared that the student took the opportunity to strengthen her understanding of the floor function by relating it to a related mathematical concept, the ceiling function.

Another example of the Task-Technology-Task* pattern occurred when students were asked to conduct a parameter exploration of the function with rule $h(x) = a \lfloor b(x-c) \rfloor + d$ where $a, b, c, d \in \Re$, and determine the effects of changing the values of $a$, $b$, $c$, and $d$.

A sketch (See Figure 4.) was provided that allowed students to change the value of a parameter and observe the corresponding changes in the graph simultaneously. To address the task of describing the effects of *d*, Rachel used the technology to decrease the value of *d* (by dragging *d* to the left) and increase the value of *d* (by dragging *d* to the right). Jim explained that changing *d* "causes it [the graph] to go up if you move it [*d*] right and [the graph moves] down if you move it [*d*] left" [Small Group Observation, 3/28, 408-409]. The researcher then asked the question, "why do you think it moves vertically? Look at the formula" [lines 416-418]. In response, Rachel posed a new task that involved connecting Jim's observation to a similar phenomenon that she may have observed when exploring linear functions. She explained that *d* is "kind of like linear…like *d* would be the constant [$b$ in a linear function $f(x) = ax + b$]" [Small Group Observation, 3/28. 424-427]. The new task that Rachel posed seemed to be an attempt to connect what she was observing in the sketch involving the floor function to a related mathematical concept.

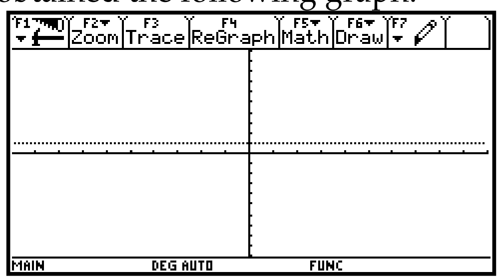
While students often were able to use technological results to make connections between the original curriculum task and other mathematical ideas, at other times the technological results puzzled students and dramatically changed the task on which they were working.

Technology as Perturbation Source

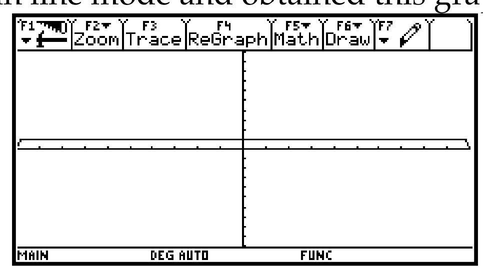
Some questions in the CAS-IM curriculum are designed to give students experience with unanticipated results of technology use. An example of this is a question that asks students to enter $ceiling(x)$ in their CAS. The TI-89 returns $-floor(-x)$. The students are then asked to reason about why $ceiling(x) = -floor(-x)$ and to use other representations such as tables and graphs to support their explanation. At other times this "technology as puzzle or perturbation source" occurs when students use the technology in exploration or corroboration. These types of situations can lead to deeper understanding of the mathematical objects or concepts under consideration as well as to developing the ability to interpret results from technology.

An example of technology as perturbation source occurred for the subjects in this study when they responded to this curriculum task: "If $g(x) = \text{ceiling}(x)$ and $h(x) = \text{floor}(x)$, what does $g(x) - h(x)$ represent? If possible write a function rule for this difference." The students had worked with the ceiling function in an earlier class session and had been introduced to the floor function on this day. They were initially given (input, output) pairs for the floor function, and in a whole class discussion they considered these ordered pairs. They were asked to generalize about the relationship between the ordered pairs, to fill in a table of values, and in a contextualized setting, asked why a business would rather use a ceiling function as opposed to a floor function to compute cell phone charges. Jim took on the task by first evaluating the floor function at $x = 2$ in his head, and then generalizing for input values that are "whole numbers" then $g(x) - h(x)$ will be 0, and if you have "like decimals, not whole numbers…then it will just be one" [Small Group Observation, 3/28,758-763].

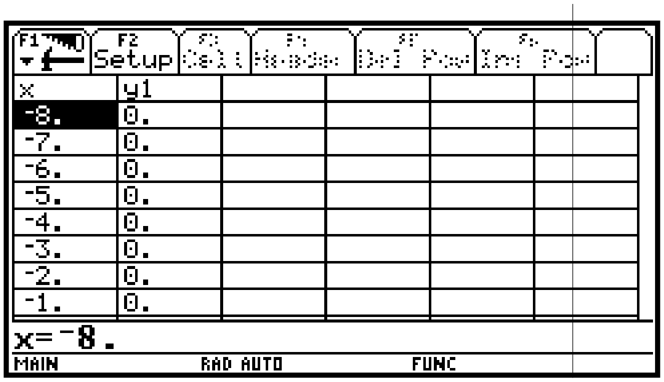
The researcher then interjected the question, "Does that make sense? Can you show me on the calculator …" [Small Group Observation, 3/28, 765-766]. It appears that this request for technology use might have been to make Jim's reasoning available to the other group members. Jim evaluated the expressions *ceiling*(2) – *floor*(2) and *ceiling* (2.3) – *floor* (2.3). The researcher then asked, "What do you think would happen if you put in ceiling of *x* minus floor of *x*? Before you actually press the button, what do you think would happen?" [Small Group Observation, 3/28, 778-780]. This question has the potential to lead to "Technology as a perturbation source" since the TI-89 and TI-92 return: – floor(–*x*) – floor(*x*). However, one student had entered the expression *ceiling*(*x*) – *floor*(*x*) into her calculator, but because something had previously been assigned to *x*, received an error message of "non-algebraic variable expression" [Small Group Observation, 3/28, 791]. Jim decided to graph *g*(*x*) – *h*(*x*) instead and obtained the following graph.




Jim immediately said "It just equals one." The researcher responded, "Hmm. But that's not what you said, was it?" [Small Group Observation, 3/28, 798-799]. Jim next put the calculator in line mode and obtained this graph.




The researcher suggested that the students zoom in, and Jim decided to examine the graph around *x* = - 10 "to see if it does anything [there]" [Small Group Observation, 3/28, 829-830]. He then traced and said, "See, *y* always equals one, no matter what number you choose…I thought that it would equal zero?" The researcher asked, "What do you think?" and Jim responded with a question "That I was wrong?" [Small Group Observation, 3/28, 844-849]. Jim went back to the standard viewing window, traced again and now *x* took on a value of – 10 and the corresponding *y* value was 0. Jim said, "Ah…It went to zero." Jim subsequently produced the following table, with integer values as inputs, and obtained outputs that were all 0.




This was still a puzzle to Jim. The researcher attempted to engage other group members but they were not able to assist Jim. Suddenly Jim said, "Oh! I see. On the graph all the numbers that are on one are not whole numbers, they're decimals" [Small Group Observation, 3/28, 905-907]. He demonstrated with the technology what occurs while using Trace and changed the table so that some output values were 0 and some were 1.

While it is possible to view technological results as being confusing, this episode illustrates the interplay between student understandings and interpretation of technological results. Because of the development of Jim's understanding of the nature of the input/output values in earlier tasks, he was able to use these understandings to make sense of technological results. A conjecture is that this type of experience will give him insight into using technology in productive ways for corroboration tasks and bringing his understandings into play when interpreting technological results. There also is a very rich *sequence* of tasks that occurred when the subjects encountered technology as perturbation source. Our analysis reveals instances in which the student is able to maintain the level of the task as posed by the researcher, and instances when the student lowers the level to collect more examples or evidence through producing values and solutions, describing observations, and eventually making sense of the technological results through a justification.

## Partitioning and Ramping up to the Task

In the previous three patterns, students began working on the task, used the technology, and then worked on a different but related task. Another pattern related to task engagement, which generally was initiated by the teacher or researcher, seemed to occur when students appeared to reach an incorrect result or an impasse while working on the original curriculum task. This pattern was described as partitioning or breaking down the original task into subtasks and then using students' responses to these subtasks as a ramp that would enable students to return to the original curriculum task.

An example of this occurred while students were working in small groups on the curriculum task that stated, "Some CASs give - *floor*(- $x$) when the user enters *ceiling*($x$). Explain why *ceiling*($x$) = - *floor*(-$x$)." Jim entered *ceiling*($x$) in the TI-92 and it returned - *floor*(- $x$). The researcher posed a justification task to the group, asking them to explain why the equation was true. Jim responded, "I guess. I don't know. Maybe they're opposites. Except if they were opposites. I don't know" [Small Group Observation, 3/28, 932-933]. After some moments of silence, the researcher, perhaps sensing that the students were unsure how to approach the task, posed a different task. She recommended that they try to figure out how - *floor*(- *x*) would operate. Jenny stated that - *floor*(- *x*) would always be positive. Rachel claimed that the negative sign in front of *floor*(- *x*) would mean that the output would always be positive. Both students seemed uncertain whether their responses were correct, and the researcher suggested that they try specific input values. Jim entered - *floor*(2.1) into the calculator. After viewing the result, Jim provided a response to the original task, stating that "when you do the floor you round down but if you put negative of rounding down, the opposite of rounding down is rounding up, so you just round up to the number" [Small group observation, 3/28, 963-966]. Although Jim's response was not stated using precise mathematical language, he seemed to be addressing the original task posed, which was to determine why *ceiling*(*x*) = - *floor*(- *x*). While at first Jim seemed uncertain about how to approach the task, working on the task of generating an example seemed to provide him with information that allowed him to return to the original task.

It also is possible that during the course of students' work on a lengthy curriculum-presented task, a teacher may choose to pose questions to students that break such a lengthy task into a sequence of much smaller (and presumably lower-level) subtasks. Although such breaking down of a task can provide the assistance students need to eventually address a higher-level task, there also is a risk in doing so. It is possible that students may become mired in the subtasks and lose sight of and never return to the main (more complex) task.

While this example illustrates how the use of technology may have supported a student's eventual return to the original task, there were other instances when students' work on the task reached an impasse and the task was never fully addressed.

Reaching an Impasse

The pattern that resulted in the eventual abandonment of the original task was observed when students did not seem to have the appropriate mathematical understandings that would enable them to reason about the task posed.

For example, after students had been exploring the effects of changing the parameter $d$ on the graph of the function with rule $h(x) = a\lfloor b(x- c) \rfloor + d$ [See Figure Y] the researcher asked the question, "Why do you think it (the graph) moves vertical? Look at the formula there, H of X is A floor of B times X minus C and that plus D" [Small group observation, 3/28, 416-417]. Rachel used the sketch to change the value of $d$ to two. The researcher introduced a point that would allow her to more easily identify the coordinates of the point where the graph intersected the $y$-axis, and the students focused on the graph and never considered the function rule or used that to explain the effects of $d$. While technology may have distracted them from this question, their inattention to it might have been attributed to their understandings of the mathematics. This might be related to students' unfamiliarity with the function rule that was introduced to them for the first time during the class or their lack of experiences with using function rules to reason about mathematical phenomena.

## Conclusion

In settings other than small group problem solving, with different tasks, and with different technology available, how might technology influence both the patterns of students' engagement with tasks and the levels of the tasks within a sequence of related tasks with which students engage? It is anticipated that additional theoretical and empirical analyses can begin to answer this question.

## References

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Jackiw, N. (199x). Geometer's Sketchpad (Version 3.1) [Computer software]. Berkeley, CA: Key Curriculum Press.

Figure 1. Chronology of the small-group sessions and interviews.

| |
|---|
| February interview (first interview) |
| March interview (second interview) |
| Day 1 of March small-group data collection in class |
| Day 2 of March small-group data collection in class |
| April interview (third interview) |
| Day 1 of May small-group data collection in class |
| Day 2 of May small-group data collection in class |
| |

Figure 2. Examples of tasks used during the small-group observations.

1 c. Why would SKYCELL prefer to use the ceiling function rather than the floor function to compute cell phone charges? Sketch a graph of $y = \lfloor x \rfloor$ and $y = \lceil x \rceil$ and use the graphs to support your answer.

3. If $g(x)$ = ceiling(x) and $h(x)$= *floor (x)*, what does $g(x) - h(x)$ represent? If possible, write a function rule for this difference.

Figure 3. The table students were asked to complete during the first small-group observation.

| $x$ | $h(x)$ |
|---|---|
| -1.2 | |
| -0.8 | |
| 0 | |
| 0.4 | |
| 2.0 | |
| 2.4 | |

Figure 4. The graph of the function $h$ with rule $h(x) = a\lfloor b(x - c)\rfloor + d$ and lines representing values of the parameters $a$, $b$, $c$, and $d$ created using The Geometer's Sketchpad.