

# **A Two-tiered Category Perspective to Describe Purposes of Mathematics Technology Use**

**Rose Mary Zbiek**

**University of Iowa/  
The Pennsylvania State University**

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## Abstract

The point of looking seriously at technology use in mathematics classrooms is not to be an advocate of glitzy equipment and dazzling displays but to investigate carefully the ways in which students and teachers use the technology. This investigation is more difficult when students use the mathematics tools (e.g., spreadsheets, computer algebra systems, graphing utilities, and dynamic construction environments) for their personal work in intellectually lively classrooms. The goal of this paper is to present the extended view of an earlier perspective (Zbiek, 1998) for looking at the purposes with which one might use technology in mathematics. The original work on only graphics/symbolic calculators was expanded to include the broad range of mathematical tools and to capture technology use by groups of individuals. The current system may be used to plan instruction as well as to interpret student, teacher, and group work.

## Two-tiered vision

Zbiek (1998) viewed the purposes of symbolic/graphic calculator use at two levels. At a more global level, she identified five large goals that one may have when using the technology. The tools may be used for *Getting Solutions (GS)*, as in generating a solution to a specific problem. This would be the case when students are asked to find the average of a data set and they immediately enter the values into a spreadsheet or calculator and have the tool compute the arithmetic mean. Getting Solutions is in opposition to the technology used for *Checking Answers (CA)* to problems that students first solve without technology or solve through multiple technology-based methods. Examples of CA work include using a computer algebra system to solve an equation that one has already solved by-hand using the quadratic formula. Users may be *Getting Information (GI)* other than complete solutions or answers from the calculators. An example is entering  $\cos(60^\circ)$  to get a decimal approximation just as one would have gotten similar information from a table of trigonometric values decades ago. In *Delegating Work (DW)* to the tools the users may or may not be able to do the mathematical work without using the calculators. Among DW events would be fitting a curve to a set of data points without first doing the computations by hand. For less mathematical needs or for communication rather than learning purposes, the technology could be used for *Improving Presentation (IP)* of mathematical ideas. This includes the use of a computer algebra system to produce a nicely printed version of the previously completed and verified calculations within a science lab report.

Within each of these five large goal areas were several more specific purposes for using the calculators. These 18 categories represent the second level of the scheme. They were initially developed to capture the essence of symbolic/graphic calculator use

in innovative materials for calculus, precalculus, and high school algebra. Because these categories have remained in tact, discussion of them will be omitted here. The original categories are denoted by an asterisk in Table 1; additional discussion of this table follows.

### **Accommodations in revision**

The main reason for expansion of the categories was to move from a tool for discussing curriculum materials to a tool for research on student and teacher use of technology in learning and teaching mathematics. The categorical system also expanded to include mathematics tools in addition to symbolic/graphic calculators and to be respectful of curriculum and content beyond a functions approach to calculus and precalculus.

The revised vision is still a two-tiered perspective. The same five large categories remain but there have been additions to the lists of second-tier categories within some of those areas. Table 1 conveys the arrangement of the large categories and their component categories of purposes along with descriptions of those components. In additions to examples from symbolic/graphic calculator use are examples involving dynamic construction environments. The second-tier categories within each of the five large categories are loosely organized. In general they are arranged around progressively more active or more mathematical issues (e.g., *Alternative Checker* denotes coordinating two or more approaches but *Answer Checker* requires implementing only one approach to the problem), or around similar notions (e.g., putting all of the “aides” together under *Delegating Work*).

Some purposes included in Table 1 did not appear in the 1998 version. Some of these categories arose from consideration of additional types of technology. Parallel categories of *Manipulation Aide* and *Construction Aide* arose to reflect the consideration of additional types of mathematics tools. It was desirable to keep *Arithmetic Aide* as a category as many tools (e.g., *The Geometer’s Sketchpad* in its calculation screen) could be used for numerical computation. Expanding *Arithmetic Aide* or folding it into a larger category to include acts such as one-command symbolic manipulation or dynamic construction using predefined menu options seemed to mask rather than illuminate understanding of technology use. Inclusion of *Representation Generator* allowed for the purposeful production of a particular representation, as opposed to the creation of several unintentionally chosen examples (as in *Example Generator*).

Other categories entered the categorical scheme when the codes were used not exclusively for curricular issues but also to capture technology-inclusive interactions in small groups or classrooms. Incorporating *Technology as Puzzle* allowed identification

of times when the technology use induces a surprise or unexpected outcome. Adding *Work Replication* was useful in denoting sections of classroom or group work when students and teachers used technology to recap what they had done rather than to do fresh mathematical work. Similarly, *Unknown* was essential when the codes were used with classroom or small group videotapes or observations when students can be seen using technology but what they enter or choose can not be discerned. *Attention Helper* provided a way to address uses of technology to highlight mathematical ideas during communication, such as using the cursor to point to a particular object on the screen while the user talked about that object.

### **Coding use**

These categories can be applied to various data sources. In the studies reported in this collection of papers the categories have been used with transcripts of interviews, small-group work, and classroom discussions. Two-part codes have been effective. One part of the code identifies the type of technology used and the second denotes one of the 18 technology purpose categories. Among the labels used for different technology types are the following:

PP Writing with various media such as paper-and-pencil, whiteboard, or overhead

89 A TI-89 calculator

92 A TI-92 calculator

C A scientific calculator

GSP *The Geometer's Sketchpad*

PO Physical objects of various types such as fingers, hands, arms, and pencils

It is possible to have two or more forms of technology as well as two or more purposes occurring simultaneously, particularly in small group or classroom settings. This implies that two or more codes could be assigned to portions of a transcript. For example a student may use the Geometer's Sketchpad to highlight the hypotenuse of  $\triangle ABC$  to see better how its length changes while dragging the vertices of the triangle to create several examples of a right triangle. The highlighting act serves an Attention Helper (AH) purpose in the midst of an Example Generator (EG) event. Thus, some lines of this transcript would carry two codes (namely, GSP-EG and GSP-AH).

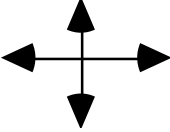
### **Conclusion**

These technology purpose codes when used with data from individual interviews, small groups and classroom discussions proved useful in discerning patterns of how students and teachers work with representations in technological settings. These codes also proved useful in investigating students' reasoning paths.

**Reference**

Zbiek, R. M. (1998). Do they really use graphics/symbolic calculators, or just hold them?  
In G. Goodell (Ed.), *Proceedings of the ninth annual international conference on  
technology in collegiate mathematics* (pp. 538-542). Reading, MA: Addison-Wesley.

**Table 1. Technology purpose categories with descriptions and examples [\* denotes category retained from 1998 version]**

	<i>Category</i>	<i>Description</i>	<i>CAS/GC Example</i>	<i>Geometry Tool Environment</i>
CA 0	Unknown UK	Technology addressed or touched with an unknown attempt.	Student is asked to match 11 expressions with 6 graphs. Student reaches for calculator; interviewer/curriculum asks student to answer the question first without using the calculator	I could use the Sketchpad to do this. (followed by no Sketchpad use and no follow-up on what the use may have been)
CA 1	Answer Checker* AC	Task is completed by hand; the same task is then done by the tool; answers are compared	Graph $y=-2x+6$ on these axes:  Use a calculator to verify your work.	Given degree measures of two angles of triangle, add and subtract their sum from 180 degrees. Use tool to construct triangle with these measures and measure third angle.
CA 2	Alternative Checker* ALC	Complete a task by hand with one method; calculator is used with a related task or different method; students compare results to determine if first answer is valid	Simplify $\frac{x^4 - 4x^3 - 8x + 32}{x^2 - 6x + 8}$ To check your answer with tool, create a table that includes columns for both the original expression and your simplified expression.	Reason that changing constant in CAS-IM logistic function rule will move graph up and down. Use slider graph with tool to check this.
GI 1	Reference Chart* RC	Obtain factual information using tool like trig/log tables, formula lists, dictionaries, etc.	Approximate $\pi$ to the nearest hundred thousandth.	Approximate $e$ to the nearest hundred thousandth.
GI 2	Information Conveyer* IC	Used as handout or overhead to provide illustrations/information to students.	Shown in a $[-5, 5, 1, -1.5, 1.5, .5]$ window are graphs of $y = \sin x$ and $y = \cos x$ . Students compare the two graphs, recalling complementary angles, to note that $\sin a = \cos b$ if $a + b = 0.5\pi$ .	Two triangles appear on screen and students determine if one can be the image of the other under a reflection.
GI 3	Technology as Puzzle TP	Tool results or process creates a result or appearance that puzzles students.	“Steps” appear on tool graph for step function family.	Dragging A of $\angle ABC$ causes degree angle measure to increase through 170s then turn negative.

DW 1	Arithmetic Aide* AA	Tool used for numerical computations as part of a larger process that itself may or may not involve technology use Notion of delegating work to the tool is essential aspect of this category.	Enter $\boxed{2}\boxed{.}\boxed{7}\boxed{\times}\boxed{8}\boxed{\wedge}\boxed{2}\boxed{=}$ while sketching by-hand graph of $y = 2.7x^2$ .	Computing $360 \div 16$ to determine the central angle measure when constructing a regular 16-gon using rotations.
DW 2	Manipulation Aide MA	Tool used for manipulation as part of a larger process that itself may or may not involve technology use. Notion of delegating work to the tool is essential aspect of this category.	Student solves quadratic equation with CAS in process of determining where graph of rational function with quadratic expression crosses the horizontal axis.	[Example may not exist]
DW 3	Construction Aide CA	Tool used for construction as part of a larger process that itself may or may not involve technology use. Notion of delegating work to the tool is essential aspect of this category.	[Example may not exist]	Student constructs horizontal line from point on slider graph to vertical axis to ascertain constant value of y for horizontal asymptote.
DW 4	Example Generator* EG	Quickly generate new instance or multiple instances from which to reason. Reasoning might involve observation and statement of patterns, followed by informal or formal explanation of why the patterns make sense.	Graph the following [using a graphics calculator]: $y = 3x^3$ , $y = 7x^3$ , $y = -2x^3$ , and $y = -4x^3$ . What effect does the sign of a have on graphs of equations of the form $y = ax^3$ ?	Dragging vertex of triangle to see that medians intersect over multiple examples of triangles.
DW 5	Representation Generator RG	Use tool to generate representation; representation may then be used in subsequent technology or non-technology work. Includes generation of an intentional case.	Use curve fitter to generate fitted function in mathematical-modeling problem-solving task.	Use script (with natural number and center point as parameters) to generate regular n-gon from which to reason about central angle measures.

DW 6	Algorithm Executor* AE	Performs tasks not (yet) doable by the user but necessary or useful in solving a problem or developing a concept	Fit various curves to data points to develop function model. What characteristics of the fitted functions (e.g., intercepts, monotonicity) match or conflict with real-world situation?	Determine image of point under circle inversion when not knowing what circle inversion is.
GS 1	Answer Giver* AG	Apply direct command to solve the stated problem	Solve $2x^2 - 8x + 9 = 17$ using direct solve command	Given three points construct the circle passing through the three points.
GS 2	Dual Processor* DP	Use two tool-based methods to solve problem or explore concept	Solve a quadratic equation with the direct solve command, by zooming with graphs, and by successive tables	Show two rectangles are congruent by measuring corresponding parts and by generating image of one under product of isometries.
IP 1	Work Replication RW	Use the tool to replicate, to reproduce, or to recover the same tool work as previously done	Reproduce a series of examples while explaining a conclusion in order to show the examples that were considered in generating the conclusion	Using a script to record a construction for the purpose of playing it back to the class
IP 2	Report Helper* RH	Aids in communicating results of investigation Includes writing summary or conclusion.	Use text editor to write up observations about a family of functions	Add text box with problem statement. Use perpendicular congruent segments with common midpoint as axes.
IP 3	Motivation Provider* MP	Use the tool to produce images or other material to illustrate or inspire a concept, process, problem, lecture or lab	Recall of graphs of $f(x) = ax^3$ , for $a = -5, -3, -1, 1, 3, \text{ and } 5$ in quick succession to illustrate the effect on the graph of increasing the value of $a$ in $f(x) = ax^3$	Show animation of dynamap to motivate study of rate of change.
IP 4	Attention Helper AH	Use the tool to draw the draw the attention of another person to some component of the display in conveying mathematics	Moving cursor to origin while saying “ This lowest point matches where the dynamap hits the 0.”	Clicks on the hypotenuse of a right triangle to show “the long side” that goes with $c$ in $a^2 + b^2 = c^2$ .