MAT 221 011 Smr 08 Quiz the Last Name: Sol.

Please show your work to receive credit. For integrals you must show all your steps carefully. Point values are in parentheses.

(5) 1. What is the equation of the plane that contains the points $\underset{P}{(1, 1, 1)}$, $\underset{Q}{(1, 3, 5)}$ and $\underset{R}{(2, -1, -1)}$?

Several versions are possible:

$\underline{u} = \overrightarrow{PQ} = \langle 0, 2, 4 \rangle$ $\quad \underline{n} = \underline{u} \times \underline{v} = \langle 4, 4, -2 \rangle$

$\underline{v}$ $\overrightarrow{PR} = \langle 1, -2, -2 \rangle$

$4x + 4y - 2z = 4(1) + 4(1) - 2(1) = 6$

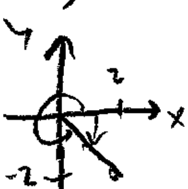
$4x + 4y - 2z = 6$

(5) 2. A point has rectangular coordinates (2, -2, -1) What are the cylindrical and spherical coordinates of the point?

Cylindrical: $r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\theta = -\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$z = -1$



so $(2\sqrt{2}, \frac{7\pi}{4}, -1)$

Spherical $\rho = \sqrt{2^2 + (-2)^2 + 1^2} = 3$

$\theta = \frac{7\pi}{4}$

$\varphi = \cos^{-1}\left(\frac{z}{\rho}\right) = \cos^{-1}\left(\frac{-1}{3}\right) \doteq 1.91\,r$

$(3, \frac{7\pi}{4}, 1.91)$

(5) 3. Let $\mathbf{r}(t) = \langle t, \sqrt{t}, t^5 \rangle$. Compute $\mathbf{a}(t) = \mathbf{r}''(t)$ and use that to find the unit vector in the direction of $\mathbf{a}(1)$.

$\mathbf{r}'(t) = \langle 1, \frac{1}{2}t^{-\frac{1}{2}}, 5t^4 \rangle$

$\underline{a}(t) = r'' = \langle 0, -\frac{1}{4}t^{-3/2}, 20t^3 \rangle$

$\underline{a}(1) = \langle 0, -\frac{1}{4}, 20 \rangle$

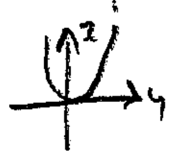
$\|\underline{a}(1)\| = \sqrt{\frac{6401}{16}} = \frac{\sqrt{6401}}{4}$

$\underline{e}_a = \langle 0, -\frac{1}{\sqrt{6401}}, \frac{80}{\sqrt{6401}} \rangle$

(5) 4. Let $\mathbf{r}(t) = \langle t^2, t, t^4 \rangle$. Carefully describe the projection of the curve onto each of the coordinate planes.

xy: $\langle t^2, t \rangle$ The parabola $x = y^2$

yz $\langle t, t^4 \rangle$ The curve $z = y^4$



xz $\langle t^2, t^4 \rangle$ The right half of $z = x^2$