MAT 221 011 Smr 08 Quiz 4 Name: **SOL**

Please show your work to receive credit. For integrals you must show all your steps carefully. Point values are in parentheses.

(5) 1. Solve the differential equation $\frac{dy}{dt} = 2\sqrt{y}$

$\frac{dy}{dt} = 2y^{1/2}$, so $y^{-1/2}dy = 2dt$ and $\int y^{-1/2}dy = \int 2dt$. Then $2y^{1/2} = 2t + C$ and $y^{1/2} = t + C$.

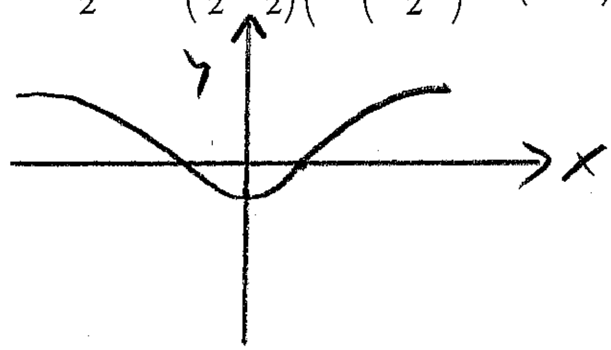
Thus $y = (t+C)^2$

(5) 2. Solve the initial value problem $y' = xe^{-y}$, $y(1) = 0$. Use your TI to sketch a graph of the solution.

$\frac{dy}{dx} = xe^{-y}$, so $e^y dy = x\,dx$ and $\int e^y dy = \int x\,dx$. Thus $e^y = \frac{x^2}{2} + C$

and $y = \ln\left(\frac{x^2}{2} + C\right)$. Since $y(1) = 0 = \ln\left(\frac{1}{2} + C\right)$

$\frac{1}{2} + C = 1$, so $C = \frac{1}{2}$ $y = \ln\left(\frac{x^2}{2} + \frac{1}{2}\right)\left(= \ln\left(\frac{x^2+1}{2}\right) = \ln(x^2+1) - \ln 2\right)$



(3 ea.) 3. Let $\mathbf{v} = \langle 1,1 \rangle$ and $\mathbf{w} = \langle 2,3 \rangle$. Calculate :

a) $\|\mathbf{w}\| = \sqrt{2^2 + 3^2} = \sqrt{13}$

b) the unit vector in the direction of $\mathbf{v} + \mathbf{w}$

$\mathbf{v} + \mathbf{w} = \langle 1+2, 1+3 \rangle = \langle 3,4 \rangle$, so $\|\mathbf{v} + \mathbf{w}\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

and $\mathbf{e}_{\mathbf{v}+\mathbf{w}} = \frac{\mathbf{v} + \mathbf{w}}{\|\mathbf{v} + \mathbf{w}\|} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

(4) 4. If the position vector for $\overrightarrow{PQ}$ is $\mathbf{v} = \langle 1,3 \rangle$ and $Q = (2,7)$, what is P?

P must be $(2-1, 7-3) = (1,4)$