MAT 221 011 Smr 08 Quiz 3 Name: Sol

Please show your work to receive credit. For integrals you must show all your steps carefully. Point values are in parentheses.

(3 ea.) 1. The polar coordinates of a point are $\left(-3, \pi/7\right)$

a) For what θ is the point also given by $(3,\theta)$? Is θ unique? Explain.

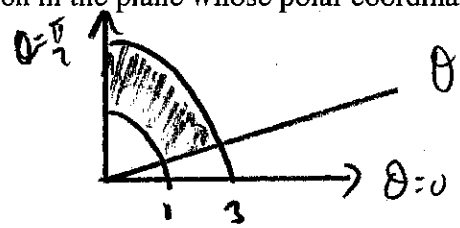
$\theta = \frac{\pi}{7} + \pi = \frac{8\pi}{7}$ will work so will $\frac{8\pi}{7} + 2k\pi$ for any whole number k. Thus θ is not unique

b) What are the (approximate) Cartesian coordinates of the point?

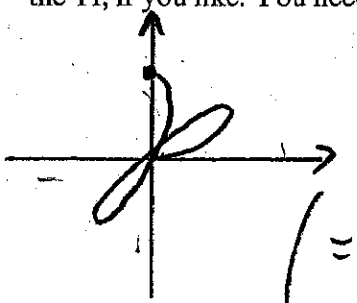
$x = r\cos\theta = -3\cos\frac{\pi}{7} \approx -2.703$

$y = r\sin\theta = -3\sin\frac{\pi}{7} \approx -1.302$ so $\left(-2.703, -1.302\right)$

(4) 2. Sketch the region in the plane whose polar coordinates satisfy the inequalities $1 \le r \le 3$, and $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$.



(5) 3. Express the arclength of $r = \sin(5\theta)$ for $0 \le \theta \le \pi/2$ as an integral. Sketch the curve, using the TI, if you like. You need not evaluate the integral.

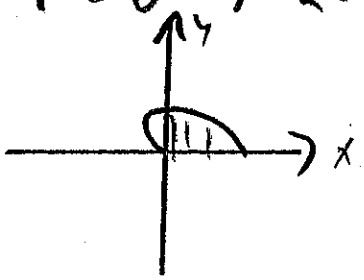


$L = \int_0^{\frac{\pi}{2}} \sqrt{(\sin 5\theta)^2 + (5\cos 5\theta)^2}\, d\theta$

$\left(= \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 5\theta + 25\cos^2 5\theta}\, d\theta \approx 5.25 \right)$

(5) 4. Consider the polar curve $r = 7 - 2\theta$ from $\theta = 0$ to the value of θ at which the curve passes through the origin. Sketch the region, using the TI, if you like. Set up and solve the integral that gives the area inside the curve on that region. Show your integration steps explicitly.

$r = 0 = 7 - 2\theta$ when $\theta = \frac{7}{2}$ $\quad A = \frac{1}{2}\int_0^{7/2} (7-2\theta)^2\, d\theta$



$= \frac{1}{2}\int_0^{7/2} 49 - 28\theta + 4\theta^2\, d\theta$

$= \frac{1}{2}\left(49\theta - 14\theta^2 + \frac{4}{3}\theta^3\right)\Big|_0^{7/2}$

$= \frac{343}{12} \approx 28.58$