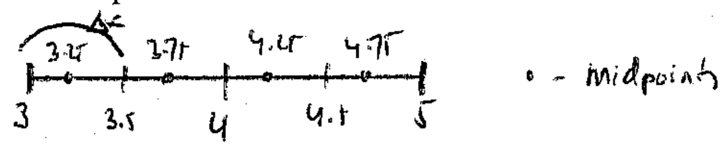
MAT 221 011 Smr 08 Quiz 2 Name: SOL

Please show your work to receive credit. For integrals you must show all your steps carefully. Point values are in parentheses.

(3 ea.) 1. For an unspecified function $f(x)$ you wish to approximate the integral $\int_3^5 f(x)\,dx$. Write out carefully what you will need to add up to calculate:

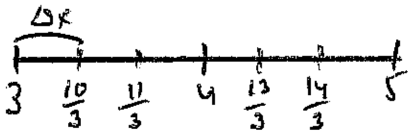
a) using the midpoint rule, M_4



$$\Delta x = \frac{5-3}{4} = \frac{1}{2}$$

so $M_4 = \frac{1}{2}\left(f(3.25) + f(3.75) + f(4.25) + f(4.75)\right)$

b) using Simpson's Rule, S_6



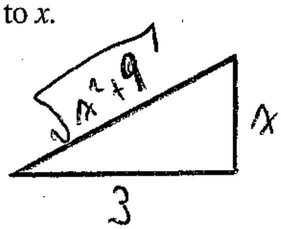
$$\Delta x = \frac{5-3}{6} = \frac{1}{3}$$

so $S_6 = \frac{1}{9}\left(f(3) + 4f(\frac{10}{3}) + 2f(\frac{11}{3}) + 4f(4) + 2f(\frac{13}{3}) + 4f(\frac{14}{3}) + f(5)\right)$

(4) 2. Set up the integral needed to find the arclength along the curve $y = x^4$ from $x = 2$ to $x = 5$. don not attempt to evaluate the integral.

Since $f'(x) = 4x^3$, $L = \int_2^5 \sqrt{1 + (4x^3)^2}\,dx = \int_2^5 \sqrt{1 + 16x^6}\,dx$

(5) 3. You have worked on an integral with the substitution $x = 3\tan\theta$ and have gotten to $\theta + \tan^2\theta\sec\theta + \sin\theta + C$. Finish the problem by converting entirely to *x*.



Since $\tan\theta = \frac{x}{3}$, we have $\theta = \tan^{-1}\frac{x}{3}$, $\sec\theta = \frac{\sqrt{x^2+9}}{3}$,

and $\sin\theta = \frac{x}{\sqrt{x^2+9}}$.

So we get $\tan^{-1}\frac{x}{3} + \frac{x^2\sqrt{x^2+9}}{27} + \frac{x}{\sqrt{x^2+9}} + C$

(5) 4. Tell exactly what substitution you would use for $\int \frac{x^2}{\sqrt{4x^2+16}}\,dx$, convert the integral entirely to θ, simplify, then stop.

Let $2x = 4\tan\theta$, so $x = 2\tan\theta$ and $dx = 2\sec^2\theta\,d\theta$. the integral becomes

$$\int \frac{(2\tan\theta)^2}{\sqrt{16\tan^2\theta + 16}} 2\sec^2\theta\,d\theta = \int \frac{4\tan^2\theta}{4\sec\theta} 2\sec^2\theta\,d\theta = 2\int \tan^2\theta\sec\theta\,d\theta$$

$(= 2\int(\sec^2\theta - 1)\sec\theta\,d\theta = 2\int \sec^3\theta - \sec\theta\,d\theta$, which is not too bad!)