MAT 221 011 Smr 08 Exam 2 June 24

Work on the lined paper. Please show your work to receive credit. Read directions carefully. Point values are in parentheses.

(6 ea.) 1. a) Give polar coordinates for the points in $xy$ coordinates $P=(3,3)$ and $Q=(2,1)$

b) The polar coordinates of a point are $\left(-3,-\pi/3\right)$. Give alternative polar coordinates for this point with both $r$ and $\theta$ positive.

(8) 2. Use your TI to graph $r=2$ and $r=4\sin(3\theta)$ together on the same screen. Sketch the graph on your paper. Now set up, but do not solve the integral needed to find the area in the first quadrant that lies inside $r=4\sin(3\theta)$ and outside $r=2$.

(8) 3. Set up the integral needed to find the distance (arclength) once around the curve given by $r=3\cos\theta$. Use your TI to find the value of the integral. This is a well-known curve. What is it and how does that show you that the answer to the integral is correct?

(8) 4. Solve the differential equation $e^{-x}y'=y$.

(8) 5. Solve the initial value problem $y'=\dfrac{x^2}{y},\ y(3)=-\sqrt{10}$.

(7) 6. Let $\mathbf{v}=\langle 3,-2,1\rangle$. What is $\|\mathbf{v}\|$ ? Give the unit vector in the direction of $\mathbf{v}$.

(7) 7. What is the angle, in decimal radians, between the vectors $\mathbf{a}=\langle 1,2,-3\rangle$ and $\mathbf{b}=\langle 0,2,-3\rangle$?

(7) 8. Give vector and parametric equations for the line through the points $P(1,-2,2)$ and $Q(-1,3,4)$

(7) 9. Let $\mathbf{v}=\langle 3,y,1\rangle$ and $\mathbf{w}=\langle 1,-2,3\rangle$. What value of $y$ will make $\mathbf{v}$ orthogonal to $\mathbf{w}$?

(7) 10.. Find the vector $\mathbf{proj_a b}$, the projection of $\mathbf{b}=\langle 1,0,2\rangle$ onto $\mathbf{a}=\langle 1,3,1\rangle$.

(7) 11. Let $\mathbf{u}=\langle -1,3,0\rangle$ and $\mathbf{v}=\langle 1,2,0\rangle$ be two vectors in the $xy$-plane. Carefully calculate $\mathbf{u}\times\mathbf{v}$ by hand.

(7) 12. What is the area of the parallelogram spanned by $\langle 1,1,3\rangle$ and $\langle 1,0,-1\rangle$?

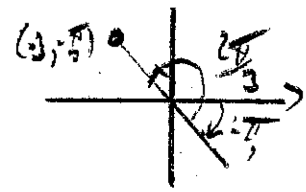
(7) 13. A line goes through the points $P(1,1,-1)$ and $Q(-1,3,2)$. What is a direction vector $\mathbf{v}$ for that line? Another line has vector equation $\mathbf{r}(t)=\langle 1+4t,3-4t,4+k\,t\rangle$. What value should $k$ be given to to make the two lines parallel?

MAT 221 EXAM 2 Sol

(6 ea.) 1. a) $P(3,3)$: $r=\sqrt{9+9}=3\sqrt{2}$ $\theta=\frac{\pi}{4}$, so $(3\sqrt{2},\frac{\pi}{4})$

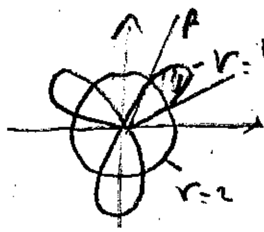
$Q(2,1)$: $r=\sqrt{4+1}=\sqrt{5}$ $\tan\theta=\frac{1}{2}$ so $\theta=\tan^{-1}(\frac{1}{2})\approx .464$

$(2.236, .464)$



b) $(3,\frac{2\pi}{3})$ or $(3,\frac{2\pi}{3}+2k\pi)$

(8) 2.



$A=\int_\alpha^\beta \frac{1}{2}(4\sin 3\theta)^2-2^2\,d\theta$

To find $\alpha$ & $\beta$ $4\sin 3\theta=2$

$\sin 3\theta=\frac{1}{2}$ so $3\theta=\frac{\pi}{6}$ $\theta=\frac{\pi}{18}$

Thus $\alpha=\frac{\pi}{18}(\approx .171)$ $\beta=\frac{\pi}{6}-\frac{\pi}{18}=\frac{5\pi}{18}(\approx .873)$

$A=\frac{1}{2}\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} 16\sin^2 3\theta-4\,d\theta$

(8) 3. Once around is $0\le\theta\le\pi$ $r'(\theta)=-3\sin\theta$

$L=\int_0^\pi\sqrt{(3\cos\theta)^2+(-3\sin\theta)^2}\,d\theta=\int_0^\pi\sqrt{9(\cos^2\theta+\sin^2\theta)}\,d\theta$

$=3\pi$. This is a circle of radius $\frac{3}{2}$, whose circumference is $2\pi(\frac{3}{2})=3\pi$

(8) 4. $\frac{1}{y}\frac{dy}{dx}=e^x$ so $\int\frac{1}{y}\,dy=\int e^x\,dx$

$\ln y=e^x+C$

$y=e^{(e^x+C)}=e^Ce^{e^x}$ $(y=De^{e^x})$

(8) 5. $y\,dy = x^2\,dx$ so $\int y\,dy = \int x^2\,dx$

$\frac{y^2}{2} = \frac{x^3}{3} + C$ so $y^2 = \frac{2}{3}x^3 + C$ & $y = \pm\sqrt{\frac{2}{3}x^3 + c}$

When $y(3) = -\sqrt{10} = -\sqrt{\frac{2}{3}(3)^3 + C} = -\sqrt{18+c}$

so $C = -8$ and $y = -\sqrt{\frac{2}{3}x^3 - 8}$

(7) 6. $\|\underline{v}\| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$ $(\approx 3.742)$

The unit vector in the direction of $\underline{v} = \underline{e}_{\underline{v}} = \left\langle \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right\rangle$

(7) 7. $\underline{a} \cdot \underline{b} = \|\underline{a}\|\|\underline{b}\| \cos\theta$

$13 = \sqrt{14}\sqrt{13}\cos\theta$

so $\theta = \cos^{-1}\left(\frac{13}{\sqrt{182}}\right) \approx .271$ rad.

(7) 8. $\underline{v} = \overrightarrow{PQ} = \langle -2, 5, 2 \rangle$

so $\underline{r}(t) = \langle 1, -2, 2 \rangle + t\langle -2, 5, 2 \rangle$

so $x = 1 - 2t$, $y = -2 + 5t$, $z = 2 + 2t$

(7) 9. We need $0 = \underline{u} \cdot \underline{v} = 3 - 2y + 3$ so $2y = 6$ & $y = 3$.

(7) 10. $\text{proj}_{\underline{a}}\,\underline{b} = \left(\frac{\underline{a} \cdot \underline{b}}{\underline{a} \cdot \underline{a}}\right)\underline{a} = \frac{3}{11}\langle 1, 3, 1 \rangle = \left\langle \frac{3}{11}, \frac{9}{11}, \frac{3}{11} \right\rangle$

(7) 11. $\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 3 & 0 \\ 1 & 2 & 0 \end{vmatrix} = \underline{i}\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} - \underline{j}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \underline{k}\begin{vmatrix} -1 & 3 \\ 1 & 2 \end{vmatrix}$

$= 0\,\underline{i} + 0\,\underline{j} - 5\,\underline{k} = \langle 0, 0, -5 \rangle$

(7) 12. $A = \| \underline{v} \times \underline{w} \| = \| \langle -1, 4, -1 \rangle \| = \sqrt{1+16+1} = \sqrt{18}$

$A \approx 4.243$

(7) 13. $\underline{v} = \overline{PQ} = \langle -2, 2, 3 \rangle$ is one

The other line has direction $\underline{v}_2 = \langle 4, -4, k \rangle$

For the lines to be parallel $\underline{v}_2 = \lambda \underline{v}$ for some

number $\lambda$. Comparing $x$ & $y$ coordinates, $\lambda = -2$

so $k = -2 \cdot 3 = -6$.