#1 No.

$$\begin{bmatrix} 2 \\ 3 \\ 2*3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \quad \text{is in } W$$

$$5\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 30 \end{bmatrix} \quad \text{not in } W$$

$30 \neq 10 * 15$

#2 $$\begin{bmatrix} x_1 \\ x_2 \\ x_1 + 2x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span $\Rightarrow$ subspace

#3 $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

$\xrightarrow{\text{rref}}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Yes. Pivot in every row.

#4 rref is $\begin{bmatrix} \textcircled{1} & 2 & 0 & -1 & 0 \\ 0 & 0 & \textcircled{1} & -3 & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

#4a $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

#4d $\dim \text{Row}(A) = 3$

#4b $x_1 = -2x_2 + x_4$

$x_2 = x_2$

$x_3 = 3x_4$

$x_4 = x_4$

$x_5 = 0$

$x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

basis for $\text{Nul}(A)$

#4f $\dim \text{Nul}(A) = 2$

#4c $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

#4e $\dim \text{Col}(A) = 3$

#4g $\text{Nul}(A) \subseteq \mathbb{R}^5$ #4h $\text{Col}(A) \subseteq \mathbb{R}^4$

#5 rref is $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

#5a $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}$

#5d $\dim \text{Row}(A) = 2$

#5b $x_1 = -3x_3$, $x_2 = 2x_3$, $x_3 = x_3$

$x_3 \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$

basis for $\text{Nul}(A)$

#5f $\dim \text{Nul}(A) = 1$

#5c $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

#5e $\dim \text{Col}(A) = 2$

#5g $\text{Nul}(A) \subseteq \mathbb{R}^3$

#5h $\text{Col}(A) \subseteq \mathbb{R}^4$

#6 $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 5 & 1 \\ -2 & -1 & 1 \end{bmatrix}$

$\xrightarrow{\text{rref}}$

$\begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{bmatrix}$

NOT LI Free variable

#7
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

$\xrightarrow{\text{rref}}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

yes rref is I_4.

#8 $\text{rank}(A) + \dim \text{Null}(A) = 7$

$\Downarrow$

3

#8a $\text{rank}(A) = 4 = \dim \text{Row}(A)$

$\Downarrow$

#8b one zero row $= \{[0\ 0\ 0\ 0\ 0\ 0\ 0]\}$ row rank $= 4$

5 rows
$4 =$ row rank
1 row of 0s

$\dim \text{Null}(A) = 3$

$\Downarrow$

#8c 3 free variables

$\Downarrow$

4 pivots

#9 $x_1\begin{bmatrix}1\\1\end{bmatrix} + x_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}7\\2\end{bmatrix}$

$$\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}7\\2\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 7\\ 1 & 1 & 2\end{bmatrix}$$

$\sim$ rref

$$\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & 5\end{bmatrix}$$

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}-3\\5\end{bmatrix}$$

#10a $2 \leq$ # vectors $< \dim \mathbb{R}^3 = 3$

$\Downarrow$

can't span

#10b $4 \leq$ # vectors $> \dim \mathbb{R}^3 = 3$

$\Downarrow$

can't be LI