

$$\#1a \quad x_1 \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$\#1b \quad \begin{bmatrix} -2 & 0 & 4 & -1 & 5 \\ 2 & 1 & -1 & 0 & -4 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$\#2a \quad x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

ref

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$x_1 = 2 \quad x_2 = 1 \quad x_3 = -2$$

#2b Yes. Pivot in every row,

#3a

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & -2 & 2 \\ 2 & -1 & 5 & 4 \\ 2 & 1 & 3 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

linearly dependent - free variable

#3b No.

$$\#4a \quad \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

#4b

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix}$$

$$T^{-1}(x_1, x_2, x_3) = (-x_1 + x_2, -x_1 + x_3, 6x_1 - 2x_2 - 3x_3)$$

#5

rref

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

#5a Yes, pivot in each row

#5b free variable \Rightarrow NOT one-to-one

#6 rref of augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$0 = 1$$

no solution

#7A

ref of coefficient matrix

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$x_1 = -2x_4$$

$$x_2 = -x_4$$

$$x_3 = x_4$$

x_4 is free

#8a

$$\det \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix} = \sin^2 \theta + \cos^2 \theta = 1$$

$\det \neq 0 \Rightarrow$ invertible

#8b

$$\begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$$

#9

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A B