

MAT 234 – 001
Fall 2009
Test One

1. Consider the following system of linear equations:

$$\begin{array}{rccccrcr} -2x_1 & & +4x_3 & -x_4 & 5x_5 & = & -2 \\ 2x_1 & +x_2 & -x_3 & & -4x_5 & = & -1 \\ x_1 & +x_2 & +x_3 & +x_4 & & = & 1 \end{array}$$

1a. Write a vector equation that is equivalent to the system.

1b. Write a matrix equation that is equivalent to the system.

2a. Write $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of the vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

2b. Can every vector in \mathbb{R}^3 be expressed as a linear combination of the vectors in $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$? Explain.

3a. Determine whether that the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \right\}$ is linearly independent or linearly dependent.

3b. Does the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \right\}$ span \mathbb{R}^4 ?

4. $T(x_1, x_2, x_3) = (2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + x_3, 2x_1 + 4x_2 + x_3)$.

4a. Write the standard matrix of T .

4b. Find T^{-1} .

5. T is the linear transformation with standard matrix

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \\ 1 & 1 & 2 & 2 & 5 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

5a. Is T onto? Explain.

5b. Is T one-to-one? Explain.

6. Describe the solutions of the following system of linear equations:

$$\begin{aligned}x - y + 2z &= 4 \\x \quad \quad z &= 6 \\2x - 3y + 5z &= 4 \\3x + 2y - z &= 1\end{aligned}$$

7. Describe the solutions of the following system of linear equations:

$$\begin{aligned}x + 2y + z + 3w &= 0 \\x - y \quad \quad + w &= 0 \\y - z + 2w &= 0\end{aligned}$$

8a. Show that the matrix $\begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$ is invertible.

8b. Find its inverse.

9. The matrix product AB is defined. If the first row of A contains all zeros, what can be said about the first row of AB ? Explain.