

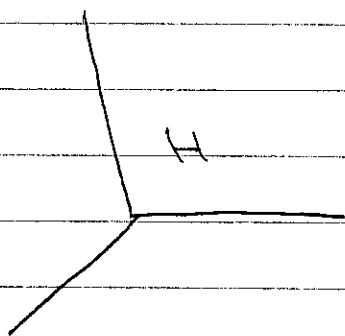
#1

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

\Downarrow
subspace

#2



$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in H$$

$$-1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$\notin H$

#3

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

row ref

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

columns span \mathbb{R}^3

#4

ref of A

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

#4a

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$$

#4b

columns 1, 2, and 4 of A

$$\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

#4c

$$x_1 + 2x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_4 = 0$$

$$x_1 = -2x_3$$

$$x_2 = x_3$$

$$x_3 = x_3$$

$$x_4 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\#4d \quad 3$$

$$\#4e \quad 3$$

$$\#4f \quad 1$$

$$\#4g \quad \mathbb{R}^4$$

$$\#4h \quad \mathbb{R}^4$$

#5 A is in rref

$$\begin{aligned} \#5a \quad x_1 + 2x_2 - x_5 + 5x_7 &= 0 \\ x_3 + x_5 - 4x_7 &= 0 \\ x_4 - x_5 - 3x_7 &= 0 \\ x_6 + 2x_7 &= 0 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -2x_2 + x_5 - 5x_7 \\ x_2 \\ -x_5 + 4x_7 \\ x_5 + 3x_7 \\ x_5 \\ -2x_7 \\ x_7 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_7 \begin{bmatrix} -5 \\ 4 \\ 3 \\ -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\dim \text{Nul } A = 3$$

#5b basis for Col A = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

dim Col A = 4

#5c basis for Row A = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

dim Row A = 4

#6

$$\begin{bmatrix} -1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

\rightarrow rref

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

linearly independent

#7

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -2 & 2 \\ 2 & -1 & 0 & 4 \\ 2 & 1 & 3 & 5 \end{bmatrix}$$

\rightarrow rref

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

NOT basis

