#1a

$$x_1\begin{bmatrix}2\\3\\1\\6\\5\end{bmatrix}+x_2\begin{bmatrix}1\\-2\\1\\0\\-1\end{bmatrix}+x_3\begin{bmatrix}1\\1\\-1\\1\\2\end{bmatrix}+x_4\begin{bmatrix}-2\\-6\\-1\\-9\\-8\end{bmatrix}=\begin{bmatrix}1\\-2\\-1\\-2\\3\end{bmatrix}$$

#1b

$$\begin{bmatrix}2&1&1&-2\\3&-2&1&-6\\1&1&-1&-1\\6&0&1&-9\\5&-1&2&-8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}1\\-2\\-1\\-2\\3\end{bmatrix}$$

#1c

rref

$$\begin{bmatrix}1&0&0&-17/11&0\\0&1&0&9/11&0\\0&0&1&3/11&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$$

$0=1$

inconsistent

#2

$$x_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+x_2\begin{bmatrix}6\\4\\2\end{bmatrix}=\begin{bmatrix}9\\2\\7\end{bmatrix}$$

$$\begin{bmatrix}1&6\\2&4\\-1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}9\\2\\7\end{bmatrix}$$

$$\begin{bmatrix}1&6&9\\2&4&2\\-1&2&7\end{bmatrix}\overset{\text{rref}}{\rightsquigarrow}\begin{bmatrix}1&0&-3\\0&1&2\\0&0&0\end{bmatrix}$$

$$x_1=-3 \quad x_2=2$$

#3

$$\begin{bmatrix}-3&5&1\\0&-1&1\\4&2&3\end{bmatrix}\overset{\text{rref}}{\rightsquigarrow}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

columns are linearly independent

#4a

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

#4b

$$A^{-1} = \begin{bmatrix} 4 & -2 & -3 \\ -11 & 6 & 9 \\ -12 & 7 & 10 \end{bmatrix}$$

$$T^{-1}(x_1, x_2, x_3) = (4x_1 - 2x_2 - 3x_3, -11x_1 + 6x_2 + 9x_3, -12x_1 + 7x_2 + 10x_3)$$

#5

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \overset{\text{rref}}{\sim} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

onto - columns span $\mathbb{R}^2$

NOT one-to-one - columns are linearly dependent

#6

$$\begin{bmatrix} 1 & -3 \\ -1 & 2 \\ 2 & -1 \end{bmatrix} \overset{\text{rref}}{\sim} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

NOT onto - columns do not span $\mathbb{R}^3$

one-to-one - columns are linearly independent

#7 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

#8 $Ax = Ay$

$Ax - Ay = 0$

$A(x-y) = 0$

$x - y \neq 0$

homogeneous system has more than just trivial solution

$\Rightarrow A$ is singular

#9 $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 2 & 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Big\downarrow$ rref

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 - x_3 = 0$ $\quad$ $x_1 = x_3$

$x_2 + x_3 = 0$ $\quad$ $x_2 = -x_3$ $\quad = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$x_3 = x_3$ free

#10

$$(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$$

$$\left[(7A)^{-1}\right]^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}^{-1}$$

$$7A = \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2/7 & 1 \\ 1/7 & 3/7 \end{bmatrix}$$