

MAT 234 – 001
Fall 2009
Review for Test One

1. Consider the following system of linear equations:

$$2x_1 + x_2 + x_3 - 2x_4 = 1$$

$$3x_1 - 2x_2 + x_3 - 6x_4 = -2$$

$$x_1 + x_2 - x_3 - x_4 = -1$$

$$6x_1 + x_3 - 9x_4 = -2$$

$$5x_1 - x_2 + 2x_3 - 8x_4 = 3$$

1a. Write a vector equation that is equivalent to the system.

1b. Write a matrix equation that is equivalent to the system.

1c. Determine whether the system of equations is inconsistent, has a unique solution, or has infinitely many solutions.

2. Write $\begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$.

3. Determine whether that the set $\left\{ \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$ is linearly independent or linearly dependent.

4. $T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3)$.

4a. Write the standard matrix of T .

4b. Find T^{-1} .

5. T is the linear transformation with standard matrix $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$.

5a. Is T onto? Explain.

5b. Is T one-to-one? Explain.

6. T is the linear transformation with standard matrix $\begin{bmatrix} 1 & -3 \\ -1 & 2 \\ 2 & -1 \end{bmatrix}$.

6a. Is T onto? Explain.

6b. Is T one-to-one? Explain.

7. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a linear transformation.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 + x_3 \end{bmatrix}$$

Write the standard matrix for T .

8. A is an $n \times n$ matrix. u and v are vectors, and $u \neq v$. But, $Ax = Ay$. Show that A must be singular.

9. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$. The standard matrix of T is

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 2 & 6 & 4 \end{bmatrix}$$

Describe the vectors that are transformed to the $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ by T .

10. A is an invertible matrix. $(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$. Determine A .