

MAT 234 - 001
Fall 2009
Test Two

Show all work.

1. Is $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_1 x_2 \end{bmatrix} \right\}$ a subspace of \mathbb{R}^3 ? Explain.

2. Is $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_1 + 2x_2 \end{bmatrix} \right\}$ a subspace of \mathbb{R}^3 ? Explain.

3. Does $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ span \mathbb{R}^3 ? Explain.

4. $A = \begin{bmatrix} 1 & 2 & 1 & -4 & 1 \\ 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -5 & 0 \\ 1 & 2 & 3 & -10 & 1 \end{bmatrix}$

4a. Find a basis for $Row(A)$.

4b. Find a basis for $Nul(A)$.

4c. Find a basis for $Col(A)$.

4d. Find the dimension of $Row(A)$.

4e. Find the dimension of $Col(A)$.

4f. Find the dimension of $Nul(A)$.

4g. $Nul(A) \subseteq \mathbb{R}^n$ for what value of n ?

4h. $Col(A) \subseteq \mathbb{R}^m$ for what value of m ?

5. $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

5a. Find a basis for $Row(A)$.

5b. Find a basis for $Nul(A)$.

5c. Find a basis for $Col(A)$.

5d. Find the dimension of $Row(A)$.

5e. Find the dimension of $Col(A)$.

5f. Find the dimension of $Nul(A)$.

5g. $Nul(A) \subseteq \mathbb{R}^n$ for what value of n ?

5h. $Col(A) \subseteq \mathbb{R}^m$ for what value of m ?

6. Is the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ linearly independent? Explain.

7. Is the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ a basis for \mathbb{R}^4 ? Explain.

8. A is a 5×7 matrix with $\dim Nul(A) = 3$.

8a. What is the rank of A ?

8b. The row reduced echelon form of A has how many zero rows?

8c. The homogeneous system of equations with the entries of A as coefficients has how many pivots? How many free variables?

9. Find the coordinates of $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ in terms of the basis $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ for \mathbb{R}^2 .

10a. Explain why $\left\{ \begin{bmatrix} 7 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix} \right\}$ cannot be a basis for \mathbb{R}^3 .

10b. Explain why $\left\{ \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 13 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 9 \end{bmatrix} \right\}$ cannot be a basis for \mathbb{R}^3 .