

MAT 234 - 001  
Fall 2009  
Review for Test Two

**Show all work.**

1. Is  $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \right\}$  a subspace of  $\mathbb{R}^3$ ? Explain.

2. Show that  $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 \geq 0 \text{ and } x_2 \geq 0 \text{ and } x_3 \geq 0 \right\}$  is NOT a subspace of  $\mathbb{R}^3$ .

3. Show that  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$  spans  $\mathbb{R}^3$ .

$$4. A = \begin{bmatrix} -1 & 0 & -2 & -1 \\ -1 & 1 & -3 & 0 \\ 2 & 0 & 4 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

4a. Find a basis for  $Row(A)$ .

4b. Find a basis for  $Nul(A)$ .

4c. Find a basis for  $Col(A)$ .

4d. Find the dimension of  $Row(A)$ .

4e. Find the dimension of  $Col(A)$ .

4f. Find the dimension of  $Nul(A)$ .

4g.  $Nul(A) \subseteq \mathbb{R}^n$  for what value of  $n$ ?

4h.  $Col(A) \subseteq \mathbb{R}^m$  for what value of  $m$ ?

$$5. A = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

5a. What is the dimension of  $Nul(A)$ ?

5b. What is the dimension of  $Col(A)$ ?

5c. What is the dimension of  $Row(A)$ ?

6. Is the set  $\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$  linearly independent?

7. Is the set  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \right\}$  a basis for  $\mathbb{R}^4$ ?

8. Is it possible for a  $5 \times 14$  matrix to have a null space of dimension 8? Explain.

9.  $A^3 = A$ . What are the possible values of  $\det(A)$ ?