#1

$y = x^3 - 3x^2$

$y' = 3x^2 - 6x$

Set $y' = 3x^2 - 6x = 0$

$3x(x-2) = 0$

critical points $x = 0 \quad x = 2$

-2, 5, 1
-10, 10, 1

critical points	$x=0$	$y=0$	
	$x=2$	$y=-4$	abs. min.
endpoints	$x=-1$	$y=-4$	abs. min.
	$x=4$	$y=16$	abs. Max

#2

$y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$

Set $y' = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

critical points $x = 0 \quad x = 3$

$y'' = 12x^2 - 24x$

Set $y'' = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \quad x = 2$

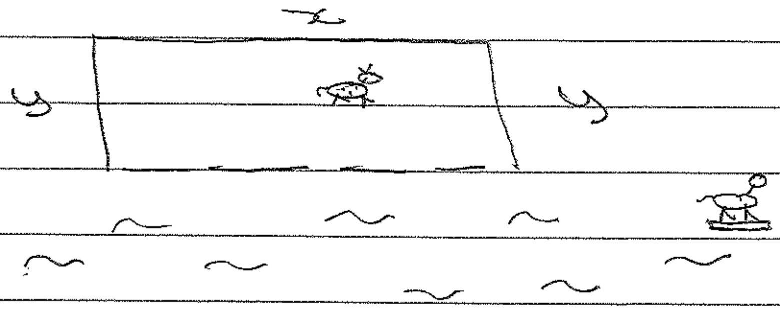
-2, 5, 1
-30, 10, 5

$x=0 \qquad x=2$

$y''>0 \quad y''=0 \quad y''<0 \quad y''=0 \quad y''>0$

ccu IP ccd IP ccu

#3



$$180000 = A = xy$$

$$F = x + 2y$$

$$F = x + 2\frac{180000}{x}$$

$$F' = 1 - \frac{360000}{x^2}$$

Set $F' = 1 - \frac{360000}{x^2} = 0$

$$x^2 = 360000$$

$$x = 600$$

$$y = \frac{180000}{600} = 300$$

$F'' = \frac{720000}{x^3} > 0$ when $x=600$ (min)

#4

$$f''(x) = x - \cos x$$

$$f'(x) = x^2/2 - \sin x + C$$

$$2 = f'(0) = 0 - \sin 0 + C$$

$$f'(x) = x^2/2 - \sin x + 2$$

$$f(x) = x^3/6 + \cos x + 2x + C$$

$$-2 = f(0) = 0 + \underbrace{\cos 0}_{1} + 0 + C$$

$$C = -3$$

$$f(x) = x^3/6 + \cos x + 2x - 3$$

subintervals $\underbrace{\qquad}_{3/n}$

#5 $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n} * \left(2 + k * 3/n\right)^4$ function

counter 1, 2, ..., n
right endpoint

$k=1$

$2 \qquad 2+3/n \qquad \cdots \qquad 2+n*3/n = 5$

$$\int_2^5 x^4 \, dx$$

#6

$2 \qquad 5$

$\frac{5-2}{6} = 1/2$

$2+0*1/2$ (2) $\quad 2+2*1/2$ (3) $\quad 2+4*1/2$ (4) $\quad 2+6*1/2 = 5$ (5)

$2 \qquad\qquad\qquad\qquad 5$

$2+1/2 \qquad 2+3*1/2 \qquad 2+5*1/2$

$5/2 \qquad 7/2 \qquad 9/2$

$$\tfrac{1}{2} f(2) + \tfrac{1}{2} f(5/2) + \tfrac{1}{2} f(3)$$
$$+ \tfrac{1}{2} f(7/2) + \tfrac{1}{2} f(4) + \tfrac{1}{2} f(9/2)$$

#7 $\int_1^3 (4x^{3/2} + x^{7/2})\,dx$

$$= \left(4\,\frac{x^{5/2}}{5/2} + \frac{x^{9/2}}{9/2}\right)\Bigg|_1^3$$

$$= \left(\frac{8}{5}x^{5/2} + \frac{2}{9}x^{9/2}\right)\Bigg|_1^3$$

$$= \left[\frac{8}{5}(3)^{5/2} + \frac{2}{9}(3)^{9/2}\right] - \left[\frac{8}{5}(1)^{5/2} - \frac{2}{9}(1)^{9/2}\right]$$

$$= \frac{162\sqrt{3}}{5} - \frac{82}{45}$$

#8 $g(x) = \int_0^{x^2} \sin^2 t\,dt$

Set $u = x^2$

$g(x) = \int_0^{u} \sin^2 t\,dt$ where $u = x^2$

Chain rule

$$\frac{dg}{dx} = \frac{dg}{du}\frac{du}{dx}$$

FTC II

$$= \sin^2 u * 2x$$

$$= \sin^2(x^2) * 2x$$

$$= 2x \sin^2(x^2)$$

#9 $\int \frac{x^3}{(x^4+1)^4}\,dx$

Let $u = x^4+1$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int u^{-4}\,du$

$= \frac{1}{4}\,\frac{u^{-3}}{-3} + C = -\frac{1}{12}\,\frac{1}{u^3} + C$

$= -\frac{1}{12}\,\frac{1}{(x^4+1)^3} + C$

#10 $\int \sin^2 x \cos x\,dx$

Let $u = \sin x$

$du = \cos x\,dx$

$\int u^2\,du = u^3/3 + C = \frac{1}{3}\sin^3 x + C$

#11 $\int_1^2 (x+1)(x^2+2x)^3\,dx$

Let $u = x^2+2x$

$du = (2x+2)\,dx$

$\frac{1}{2}du = (x+1)\,dx$

$u = x^2+2x$

$x=2 \qquad u=8$

$x=1 \qquad u=3$

$\frac{1}{2}\int_3^8 u^3\,du = \frac{1}{2}\frac{u^4}{4}\Big|_3^8$

$= \frac{1}{8}8^4 - \frac{1}{8}3^4$

#12 $\int_{-1}^2 \sqrt{5x+6}\,dx$

Let $u = 5x+6$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$u = 5x+6$

$x=2 \qquad u=16$

$x=-1 \qquad u=1$

$\frac{1}{5}\int_1^{16} \sqrt{u}\,du = \frac{1}{5}\frac{u^{3/2}}{3/2}\Big|_1^{16}$

$= \frac{2}{15}\left(16^{3/2} - 1^{3/2}\right)$