MAT 129-003

#1a $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (x+1) = 2$

#1b $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} 1/x = 1$

#1c $\lim\limits_{x \to 1} f(x)$ does not exist

#2 $\lim\limits_{x \to -4} \dfrac{x^2+5x+4}{x^2+3x-4}$

$$= \lim_{x \to -4} \frac{(x+1)\cancel{(x+4)}}{(x-1)\cancel{(x+4)}} = \frac{-3}{-5} = 3/5$$

#3 $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 6x}$

$$= \lim_{x \to 0} \cancelto{1}{\frac{\sin 4x}{4x}} \cdot \frac{4x}{1} \cdot \cancelto{1}{\frac{6x}{\sin 6x}} \cdot \frac{1}{6x} = \frac{2}{3}$$

#4 $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{[(x+h)^2 - 8(x+h) + 9] - [x^2 - 8x + 9]}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h + \cancel{9} - \cancel{x^2} + \cancel{8x} - \cancel{9}}{h}$

$= \lim_{h\to 0} (2x + h - 8) = 2x - 8$

#5 $f(x) = x^2 - \frac{1}{\sqrt[4]{x^5}} = x^2 - x^{-5/4}$

$f'(x) = 2x + 5/4\, x^{-9/4}$

#6 $f(x) = \frac{x^2 + x - 2}{x^3 + 6}$

$f'(x) = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$

$= \frac{-(x^4 + 2x^3 - 6x^2 - 12x - 6)}{(x^3+6)^2}$

#7

$$y = x^3 - x^2 - x + 1$$
$$y' = 3x^2 - 2x - 1$$

want $y' = 0$ $\quad 0 = 3x^2 - 2x - 1$

$$0 = (3x+1)(x-1)$$

$$x = -1/3 \qquad x = 1$$
$$y = 32/27 \qquad y = 0$$

#8

$$y = 1 - x^3$$
$$y' = -3x^2$$

when $x = 0$ $\quad y' = 0$

$$y - 1 = 0(x - 0)$$
$$y = 1$$

#9

$$s = t^3 - 4.5t^2 - 7t$$
$$v = 3t^2 - 9t - 7$$

when $v = 5$ $\quad 5 = 3t^2 - 9t - 7$

$$0 = 3t^2 - 9t - 12$$
$$0 = 3(t^2 - 3t - 4)$$
$$0 = 3(t-4)(t+1)$$

$$t = 4 \quad \text{or} \quad t = -1$$