MAT 415 – 001
Spring 2009
Final exam
Do 7 problems. Due no later than noon on Friday, May 8.

1. Show that a regular 30-gon is constructible.

2. Show that the ideal $\langle X-1\rangle$ in $\mathbb{Z}[X]$ is prime but not maximal.

3. Show that $\mathbb{Z}_3[X]/\langle X^2+X+1\rangle$ is NOT a field.

4. Find the multiplicative inverse of X^2+X+1 in $Z_2[X]/\langle X^3+X+1\rangle$.

5. Is X^5+X+1 irreducible in $\mathbb{Z}_2[X]$?

6. Prove that any subfield of $\mathbb{R}$ must contain $\mathbb{Q}$.

7. k is a field. $f(X)=a_0+a_1X+\ldots+a_nX^n\in k[X]$ with $a_0\neq 0$ and $a_n\neq 0$. $f(X)$ is irreducible. Show that $a_0Y^n+a_1Y^{n-1}+\ldots+a_{n-1}Y+a_n\in k[Y]$ is irreducible.

8. p is a prime. Show that in $\mathbb{Z}_p[X]$, $X^{p-1}-1=(X-1)(X-2)\cdots(X-(p-1))$.

9. Construct a field of 9 elements.

10. E and F are fields. Show that if $[E:F]=2$, then E is a splitting field over F.

11. Prove that is $\sqrt{3}$ not a linear combination of 1 and $\sqrt{2}$ over $\mathbb{Q}$.

12. Factor $X^{18}-1$ and determine $\Phi_{18}(X)$.