MAT 415 – 001
Spring 2009
Test Three
Do 7 problems.  Due Monday, April 20

1. Describe the elements of $\mathbb{Q}\left(\sqrt[3]{5}\right)$.

2. Find a basis for $\mathbb{Q}\left(\sqrt{2},\sqrt[3]{2}\right)$ over $\mathbb{Q}$.

3. Prove that $X^2-3$ is irreducible over $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

4. Show that $\sqrt{1+\sqrt{3}}$ is algebraic over the rationals.

5. Prove that it is impossible to construct a cube whose volume is 3 times the volume of a given cube.

6. $\alpha$ and $\beta$ are transcendental over $\mathbb{Q}$. Show that either $\alpha\beta$ or $\alpha+\beta$ is also transcendental over $\mathbb{Q}$.

7. Find the minimal polynomial of $\sqrt{-3}+\sqrt{2}$ over the rationals.

8. $3+4\sqrt{2}\in\mathbb{Q}\left(\sqrt{2}\right)$. $\left\{1,\ \sqrt{2}\right\}$ is a basis of $\mathbb{Q}\left(\sqrt{2}\right)$ over $\mathbb{Q}$. Find $a,b\in\mathbb{Q}$ so that $\left(3+4\sqrt{2}\right)^{-1}=a+b\sqrt{2}$.

9. $K$ is a field extension of $F$. $a\in K$. Show that $\left[F(a):F\left(a^3\right)\right]\le 3$.

10. $\left[K:F\right]$ is prime. $u\in K$ is algebraic over $F$. Show that either $F(u)=K$ or $F(u)=F$.