

MAT 415 - 001 SPRING 2009 TEST TWO

INSTRUCTIONS

Do 7 problems.

The test is due on Wednesday, April 1.

PROBLEMS

1. Factor $X^4 + 4$ in $Z/(5)[X]$.
2. Consider $Z_3[X]/(X^2 + 2X + 2)$. This is a finite field of 9 elements. Determine the product $(2x + 2)(x + 2)$ in this field.
3. Determine whether $X^3 + 4X^2 - 2X - 1$ is irreducible over the rational numbers.
4. Determine all monic irreducible quadratic and cubic polynomials over F_3 , the field of 3 elements.
5. Show that for every integer n , $X^7 + 9X^3 + (9n + 6)$ is irreducible over the rational numbers.
6. Determine whether $X^6 + 30X^5 - 15X^3 + 6X - 120$ is irreducible over the integers.
7. $a \neq 0$ is a root of $f(X) = a_0 + a_1X + \dots + a_nX^n$ in $F[X]$. Show that $1/a$ is a root of $a_n + a_{n-1}X + \dots + a_1X^{n-1} + a_0X^n$.
8. Determine whether $X^3 + 6X^2 + 5X + 25$ is irreducible over the rational numbers.
9. $f(X) = a_0 + a_1X + \dots + a_{n-1}X^{n-1} + X^n$ is a polynomial in $k[X]$. Show that the product of the roots is in k .
10. Prove or disprove: For all integers n , $X^8 + 5X^3 + 5n$ is irreducible over the rational numbers.