

MAT 302 – 001

Spring 2009

Comprehensive exam

Do 7 problems

1.  $\sqrt{6}$  is irrational.
2. Show that every prime of the form  $3n + 1$  is of the form  $6m + 1$ .
3. Show that  $\gcd(m + n, n) = \gcd(m, n)$ .
4. Prove that  $A \cup (A' \cap B) = A \cup B$ .
5. Show that “If  $A \cap B = A \cap C$ , then  $B = C$ ” is a false statement.
6. For all  $n \in \mathbb{N}$ ,  $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}$ .
7.  $a_1 = a_2 = 1$ .  $a_n = 2a_{n-1} + a_{n-2}$  for  $n > 2$ . Prove that for all  $n \in \mathbb{N}$ ,  $a_n$  is odd.
8. Prove that for every natural number  $n \geq 14$ ,  $n$  can be written as the sum of numbers each of which is a 3 or an 8.

9. Is  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \begin{cases} x & x \leq 0 \\ x^2 - x & x > 0 \end{cases}$  a bijection?

10. If the composition  $g \circ f(x)$  is one-to-one, must  $g$  be one-to-one?

11.  $n \in \mathbb{N}$  and  $a, b \in \mathbb{Z}$ . Prove that there exist an integer  $c$  such that  $a + c \equiv b \pmod{n}$ .

12.  $R = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid m = n \text{ or } m = -n\}$ . Show that  $R$  is an equivalence relation on  $\mathbb{Z}$ .

13. A relation  $R$  is defined on the integers by  $xRy$  if  $11x - 5y$  is even. Show that  $R$  is an equivalence relation.

14. Prove or disprove. Let  $n \in \mathbb{Z}$ . If  $n^3 + n$  is even, then  $n$  is even.