

MAT 302 – 001

Spring 2009

Comprehensive exam

Do 7 problems

1. $\sqrt{6}$ is irrational.
2. Show that every prime of the form $3n + 1$ is of the form $6m + 1$.
3. Show that $\gcd(m + n, n) = \gcd(m, n)$.
4. Prove that $A \cup (A' \cap B) = A \cup B$.
5. Show that “If $A \cap B = A \cap C$, then $B = C$ ” is a false statement.
6. For all $n \in \mathbb{N}$, $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}$.
7. $a_1 = a_2 = 1$. $a_n = 2a_{n-1} + a_{n-2}$ for $n > 2$. Prove that for all $n \in \mathbb{N}$, a_n is odd.
8. Prove that for every natural number $n \geq 14$, n can be written as the sum of numbers each of which is a 3 or an 8.

9. Is $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} x & x \leq 0 \\ x^2 - x & x > 0 \end{cases}$ a bijection?

10. If the composition $g \circ f(x)$ is one-to-one, must g be one-to-one?

11. $n \in \mathbb{N}$ and $a, b \in \mathbb{Z}$. Prove that there exist an integer c such that $a + c \equiv b \pmod{n}$.

12. $R = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid m = n \text{ or } m = -n\}$. Show that R is an equivalence relation on \mathbb{Z} .

13. A relation R is defined on the integers by xRy if $11x - 5y$ is even. Show that R is an equivalence relation.

14. Prove or disprove. Let $n \in \mathbb{Z}$. If $n^3 + n$ is even, then n is even.