

MAT 302 – 001

Spring 2009

Test Three

Do 5 problems in class and turn in 1 additional problem on Friday.

1. $A = \{1, 2, 3\}$. Define a relation R on A so that R is reflexive, not symmetric, and transitive.
2. S is a relation on \mathbb{N} defined by xSy if and only if 3 divides $x + y$. Prove or disprove that S is an equivalence relation on \mathbb{N} .
3. If $ca \equiv cb \pmod{n}$ and $d = \gcd(c, n)$, then $a \equiv b \pmod{\left(\frac{n}{d}\right)}$.
4. If $a \equiv b \pmod{n}$, prove that $\gcd(a, n) = \gcd(b, n)$.
5. Prove that every integer of the form $3n + 2$ has a prime factor of this form.
6. T is a relation on $\mathbb{R} \times \mathbb{R}$. $(x, y)T(z, w)$ if and only if $x + y \leq z + w$. Prove or disprove that T is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.
7. $A = \{i, -1, -i, 1\}$ where $i^2 = -1$. The relation xRy if and only if $xy = \pm 1$ is an equivalence relation. Prove this.
8. Determine the equivalence classes of the relation in problem number 7.
9. R and S are each equivalence relations on the same set A . Prove that $R \cap S$ is an equivalence relation on A .
10. A is a non-empty set. $R = A \times A$. R is a relation on A . Prove or disprove that R is an equivalence relation on A .