

MAT 302 - 001 SPRING 2009 TEST TWO

INSTRUCTIONS

Do 5 problems in class, and do 1 more outside of class to turn in on the Monday.

1. For all natural numbers n , prove that $1 + 3 + 9 + \dots + 3^n = (3^{n+1} - 1)/2$
2. For all natural numbers n , prove that $1 + 3 + 5 + \dots + (2n + 1) = (n + 1)^2$.
3. For all natural numbers n , prove that 9 divides $10^n + 3 \times 4^{n+2} + 5$.
4. For all natural numbers $n > 1$, prove that $3^n > 1 + 2^n$.
5. For all natural numbers $n \geq 5$, $(n + 1)! > 2^{n+3}$.
6. a is an integer, n is a positive integer, and $d = \gcd(a, a + n)$. Prove that d divides n .
7. Lucas sequence: $L_1 = 1, L_2 = 3$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 3$. Prove that for $n \geq 1$, $L_1 + L_2 + L_3 + \dots + L_n = L_{n+2} - 3$.
8. Fibonacci sequence: $u_1 = 1, u_2 = 1$, and $u_n = u_{n-1} + u_{n-2}$ for $n \geq 3$. Show that $u_1 + u_3 + u_5 + \dots + u_{2n-1} = u_{2n}$.