

MAT 302 – 001

Spring 2009

Review for comprehensive exam

1. $a, b,$ and c are positive integers. If $ac|bc$, then $a|b$.
2. There do not exist integers m and n such that $2m + 4n = 7$.
3. $x \in \mathbb{Z}$. Show that $x^2 + 3x + 1$ is odd.
4. $x \notin B$ and $A \subseteq B$. Prove that $x \notin A$.
5. Show that “If $A \cap C \subseteq B \cap C$, then $A \subseteq B$ ” is a false statement.
6. For all $n \in \mathbb{N}$, $n + 3 < 5n^2$.
7. For all $n \in \mathbb{N}$, $3 | (n^3 + 5n + 6)$.
8. For all $n \in \mathbb{N}$, $1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$.
9. Show that $f : \mathbb{R} \rightarrow [1, \infty)$ is onto if $f(x) = x^2 + 1$.

10. Show that $f(x) = \begin{cases} 2-x & x \leq 1 \\ \frac{1}{x} & x > 1 \end{cases}$ is one-to-one but not onto \mathbb{R} .

11. $a \equiv b \pmod{n}$. $d|a$, $d|b$, and $d|n$ where $d > 0$. Show that $\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{n}{d}}$.

12. p is a prime. $p|a^n$. Prove that $p^n|a^n$.