

MAT 302 – 001

Spring 2009

Review for comprehensive exam

1.  $a, b,$  and  $c$  are positive integers. If  $ac|bc$ , then  $a|b$ .
2. There do not exist integers  $m$  and  $n$  such that  $2m + 4n = 7$ .
3.  $x \in \mathbb{Z}$ . Show that  $x^2 + 3x + 1$  is odd.
4.  $x \notin B$  and  $A \subseteq B$ . Prove that  $x \notin A$ .
5. Show that “If  $A \cap C \subseteq B \cap C$ , then  $A \subseteq B$ ” is a false statement.
6. For all  $n \in \mathbb{N}$ ,  $n + 3 < 5n^2$ .
7. For all  $n \in \mathbb{N}$ ,  $3|(n^3 + 5n + 6)$ .
8. For all  $n \in \mathbb{N}$ ,  $1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$ .
9. Show that  $f : \mathbb{R} \rightarrow [1, \infty)$  is onto if  $f(x) = x^2 + 1$ .

10. Show that  $f(x) = \begin{cases} 2-x & x \leq 1 \\ \frac{1}{x} & x > 1 \end{cases}$  is one-to-one but not onto  $\mathbb{R}$ .

11.  $a \equiv b \pmod{n}$ .  $d|a$ ,  $d|b$ , and  $d|n$  where  $d > 0$ . Show that  $\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{n}{d}}$ .

12.  $p$  is a prime.  $p|a^n$ . Prove that  $p^n|a^n$ .