MAT 302 – 001

Spring 2009

Review for Test Three

1. Define the relation R on $\mathbb{Z}$ by $R = \{(a, b) \mid a, b \in \mathbb{Z} \text{ and } ab \geq 0\}$. Prove or disprove that R is an equivalence relation.

2. $A = \{1, 2, 3\}$. Define a relation R on A so that R is not reflexive, not symmetric, but is transitive.

3. Define the relation R on $\mathbb{Z}$ by $R = \{(a, b) \mid a, b \in \mathbb{Z} \text{ and } a + b \text{ is even}\}$. Prove that R is an equivalence relation.

4. Determine the equivalence classes of the relation in number three.

5. R is a symmetric and transitive relation on a set A. The domain of R is all of A. Show that R is reflexive on A.

6. L is a relation on a set A. L is reflexive and transitive. Define the relation R on A by xRy if and only if xLy and yLx. Prove that R is an equivalence relation.

7. Show that there are infinitely many primes of the form $4k+1$. [Hint: Assume not. Call the finitely many primes of that form $p_1, p_2, ..., p_r$. Consider $N = (2p_1 p_2 \cdots p_r)^2 + 1$.]

8. p is a prime. Prove that $(p-1)^2 = 1 \bmod p$.

9. $a = b \bmod st$. Show that $a = b \bmod s$ and $a = b \bmod t$.