

## MAT 302 - 001 SPRING 2009 REVIEW FOR TEST TWO

### INSTRUCTIONS

There will be 5 problems to do in class and 1 to turn in on the Monday after the test.

1. For all natural numbers  $n$ , prove that  $3+11+19+\dots+(8n-5) = 4n^2-n$ .
2. For all natural numbers  $n$ , prove that  $n^3 + 5n + 6$  is divisible by 3.
3. For all natural numbers  $n$ , prove that  $2^n \geq 1 + n$ .
4. For all natural numbers  $n$ , prove that  $n + 3 < 5n^2$ .
5. Lucas sequence:  $a_1 = 1$ ,  $a_2 = 3$ , and  $a_n = a_{n-1} + a_{n-2}$  for all  $n \geq 3$ . For all natural numbers  $n$ , prove that  $a_n < (7/4)^n$ .
6.  $a_1 = 11$ ,  $a_2 = 21$ , and  $a_n = 3a_{n-1} - 2a_{n-2}$  for all  $n \geq 3$ . For all natural numbers  $n$ , prove that  $a_n = 5 \times 2^n + 1$ .
7. If  $\gcd(a,b)=d$  and  $a = da'$  and  $b = db'$ , show that  $\gcd(a', b')=1$ .
8. Prove that  $\gcd(a, bc)=1$  if and only if  $\gcd(a, b)=1$  and  $\gcd(a, c)=1$ .