MAT 114 – 006
Spring 2009
Comprehensive Exam
Show all work.

1. Set up a system of linear equations to solve the following problem. You need not solve the system. Define the unknowns that you use.

Michael Perez has a total of $2000 on deposit with two savings institutions. One pays 6% per year, and the other pays 8% per year. If Michael earned $144 in interest, how much does he have on deposit in each institution?

2. The augmented matrix of a system of linear equations (with unknowns x, y, z, and u) reduces to

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

2a. Write the system of equations that corresponds to this augmented matrix.

2b. Determine whether the system has a unique solution, no solution, or infinitely many solutions.

*3. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$\begin{aligned} x \quad &+2y \quad -3z \quad = \quad 8 \\ -2x \quad &+2y \quad +z \quad = \quad 3 \\ 3x \quad &-2y \quad +8z \quad = \quad 9 \end{aligned}$$

4. Write the initial tableau for the following linear programming problem. You need not solve the problem.

Maximize $p = 20x + 30y$.

subject to the following constraints.

$$\begin{array}{rrcl} 2x & +10y & \le & 80 \\ 6x & +2y & \le & 72 \\ 3x & +2y & \ge & 6 \\ x & & \ge & 0 \\ & y & \ge & 0 \end{array}$$

5. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	3	1	2	1	0	0	0	9
t	2	3	1	0	1	0	0	8
u	1	2	3	0	0	1	0	7
	–20	–12	–18	0	0	0	1	0

6. The following is an initial tableau. Determine the pivot.

	x	*y*	*s*	*t*	*u*	*p*	
s	1	1	1	0	0	0	8
t	5	3	0	-1	0	0	21
u	1	3	0	0	-1	0	9
	-8	-5	0	0	0	1	0

7. The following is a final tableau. Determine the maximum value of *p* and the values of *x*, *y*, and *z*.

	x	*y*	*z*	*s*	*t*	*u*	*p*	
z	0	1	2	3	–1	0	0	90
x	6	3	0	–3	3	0	0	30
u	0	0	0	–9	–1	1	0	490
	0	0	0	12	0	0	3	600

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of dietary fiber, x equal the number of servings of steak, and y equal the number of servings of cream of potato soup. You need not solve the problem.

In order to finish a term paper, Erin plans to work in her room all day on Saturday and eat all her meals at a nearby deli. She plans to eat only steak sandwiches and cream of potato soup. The nutritional contents, per serving, are:

	Steak sandwich	Cream of potato soup
calories	400	200
fat	12g	8.5g
dietary fiber	9g	5g

Erin wants to restrict her daily diet to no more than 2000 calories and no more than 65g of fat. How many servings of each food should she choose to maximize dietary fiber?

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. The following is neither an initial nor a final tableau.

	x	y	s	t	u	p	
s	0	0	2	3	1	0	2
y	0	1	0	−1	0	0	10
x	2	0	0	−1	−1	0	1
	0	0	0	−5	−1	2	50

9a. Determine the values of x, y, s, t, u, and p at this stage.

9b. Is the tableau in phase I or phase II?

9c. Determine the next pivot. You need not do the pivot operation.

10. Translate the given system of linear equations into matrix form.

$$\begin{aligned} x - y &= 1 \\ x + y + 2z &= 2 \\ x + 2y + z &= 0 \end{aligned}$$

11. Find the inverse of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -2 & 3 \end{bmatrix}$.

*12. $n(S)=100$, $n(A)=28$, $n(B)=30$, $n(C)=34$, $n(A\cap B)=10$, $n(A\cap C)=10$, $n(B\cap C)=15$, and $n(A\cap B\cap C)=5$. Find $n(A'\cap B\cap C)$.

13. A warranty identification number for a certain product consists of a letter of the alphabet followed by a 5-digit number. How many numbers are possible it the first digit of the 5-digit number must be nonzero?

14. In how many ways can a 6-letter security password be formed from the letters of the alphabet if no letter is repeated?

15. A company has 5 vacancies for its executive training program. 10 males and 10 females have applied for the training program. How many sets of 5 trainees can be selected if the vacancies must be filled by 2 men and 3 women?

16. From a set of 7 mathematics books, 5 literature books, and 9 science books; in how many ways can a student select 2 from each set?

17. A survey of 100 college faculty shows that 70 own a car, 30 own a bicycle, and 10 own both. Find the probability that a faculty member chosen at random owns a car or a bicycle.

18. A creative writing class has 32 students, of whom 16 are seniors, 12 are juniors, and 4 are sophomores. 9 of the seniors, 5 of the juniors, and 2 of the sophomores are journalism majors. As student is selected at random. Find the probability that the student is:

18a. a journalism major, given that the student is a junior.

18b. not a senior, given that the student is a journalism major.

19. A group of students consists of 12 females and 19 males. 4 of the females and 6 of the males are mathematics majors. Are the events “male” and “mathematics major” independent?

20. A football team plays 60% of its games at home and 40% away. It typically wins 80% of its home games and 55% of its away games. If the team wins on a certain Saturday, what is the probability that it played at home?