MAT 114 – 006
Spring 2009
Test One
Show all work.

1. For the opening night at the Opera House, a total of 1000 tickets were sold. Front orchestra seats cost $80 each, rear orchestra seats costs $60 each, and front balcony seats cost $50 each. The combined number of tickets sold for the front orchestra and rear orchestra exceeded twice the number of front balcony tickets sold by 400. The total receipts for the performance were $62,800. Set up a system of linear equations whose solution would determine how many tickets of each type were sold. You need not solve the system.

2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$\begin{aligned} x &- 2y &+ z &= 6 \\ 2x &+ y &- 3z &= -3 \\ x &- 3y &+ 3z &= 10 \end{aligned}$$

3. For each of the following augmented matrices of systems of linear equations, determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

3a. $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

3b. $\begin{bmatrix} 1 & 0 & 0 & 1 & 1500 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 2000 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3c. $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4a. Graph the feasible set of the following set of linear inequalities.

$$\begin{aligned} x \quad &+y \quad \leq \quad 8 \\ 2x \quad &+y \quad \leq \quad 10 \\ x \quad & \quad \geq \quad 0 \\ \quad & y \quad \geq \quad 0 \end{aligned}$$

4b. Find the corner points of the feasible set.

4c. Is the feasible set bounded?

4d. By evaluating $p = 4x + 2y$ at the corners (and, if necessary, a new corner) determine the maximum value of p that satisfies the constraints.

5a. Write the initial tableau for the following standard maximum problem. You need not solve the problem.

Maximize $p = 2x + 2y + z$

subject to the following structural constraints and nonnegativity.

$$\begin{array}{rrrcl} 2x & +y & +2z & \leq & 14 \\ 2x & +4y & +z & \leq & 26 \\ x & +2y & +3z & \leq & 28 \end{array}$$

5b. Determine the first pivot. You do not need to do the pivot – just determine the row and column of the first pivot.

6. The following is neither an initial nor a final tableau.

	x	y	z	s	t	p	constants
s	0	−7	0	1	−2	0	12
x	3	4	2	0	1	0	30
p	0	7	–3	0	3	1	90

6a. Determine the values of x, y, z, and p.

6b. Determine the row and column of the next pivot. You need not do the pivot.

7. The following is a final tableau. Determine the maximum value of p and the values of x and y.

	x	y	s	t	u	p	constants
y	0	3	−1	1	0	0	18
u	0	0	−13	1	9	0	18
x	9	0	4	−1	0	0	72
p	0	0	1	1	0	1	78

8. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal profit, x equal the number of model A fax machines, and y equal the number of model B fax machines. You need not solve the problem.

National Business Machines Corporation manufactures two models of fax machines: A and B. Each model A costs $100 to make, and each model B costs $150. The profits are $30 for each model A and $40 for each model B fax machine. If the total number of fax machines demanded each month does not exceed 2500 and the company has earmarked no more than $600,000 per month for manufacturing costs, find how many units of each model National should make to maximize its monthly profits.