MAT 114 – 006
Spring 2009
Review for Comprehensive Exam
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1. Set up a system of linear equations to solve the following problem. You need not solve the system. Define the variables that you use.

> The Dandy Donut Company makes three types of donuts: cake, glazed, and jelly. Each dozen of cake donuts requires 2 eggs, 2 cups of flour, and 1 cup of sugar. Each dozen glazed donuts requires 1 egg, 2 cups of flour, and 1 cup of sugar. Each dozen jelly donuts requires 3 eggs, 3 cups of four, and 2 cups of sugar. Suppose that 100 eggs, 130 cups of flour, and 70 cups of sugar were used on a given day. How many donuts of each type were produced?

*2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system. Determine whether there is a unique solution, no solution, or infinitely many solutions. If the solution is unique, state the values of x, y, and z. If there are infinitely many solutions, solve for the pivot variables in terms of the free variables.

$$\begin{aligned} x &+2y &-3z &= 5 \\ 2x &+3y &-4z &= 7 \\ -3x &-4y &+5z &= -9 \end{aligned}$$

3. The augmented matrix of a system of linear equations (with unknowns x, y, z, and u) reduces to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

3a. Write the system of equations that corresponds to this augmented matrix.

3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4. Write the initial tableau for the following linear programing problem. You need not solve the problem.

Maximize $p = 3x - y$.

subject to the following constraints.

$$\begin{array}{rrcl} x & +2y & \le & 80 \\ -x & +3y & \le & 60 \\ x & & \le & 50 \\ x & & \ge & 0 \\ & y & \ge & 0 \end{array}$$

5. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	2	2	1	0	0	0	30
t	3	0	1	0	1	0	0	36
u	0	2	1	0	0	1	0	14
	–4	–8	–6	0	0	0	1	0

6. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	4	1	–1	–1	0	0	0	6
t	3	1	0	0	–1	0	0	5
u	2	1	0	0	0	–1	0	4
	36	12	–7	0	0	0	1	0

7. The following is a final tableau. Determine the maximum value of p and the values of x, y, and z.

	x	y	z	s	t	u	p	
y	0	6	0	1	–1	3	0	1200
x	4	0	0	1	1	–3	0	400
z	0	0	6	–1	1	3	0	900
	0	0	0	5	1	9	12	7800

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of profit, x equal the number of machine A, and y equal the number of machine B produced. You need not solve the problem.

> National Business Machines Corporation manufactures two models of fax machines: A and B. Each model A costs $100 to make, and each model B costs $150. The profits are $30 for each model A and $40 for each model B fax machine. If the total number of fax machines demanded each month does not exceed 2500 and the company has earmarked no more than $600,000 per month for manufacturing costs, find how many units of each model the company should make each month to maximize its monthly profits.

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. The following is neither an initial nor a final tableau.

	x	y	z	s	t	u	p	
s	2	0	12	4	0	–1	0	2000
t	0	0	–8	0	1	–2	0	2000
y	2	4	8	0	0	1	0	4000
	–2	0	13	0	0	2	1	8000

9a. Determine the values of x, y, z, s, t, u, and p at this stage.

9b. Is the tableau in phase I or phase II?

9c. Determine the next pivot. You need not do the pivot operation.

10. Translate the given system of linear equations into matrix form.

$$\begin{array}{rrrcr} 2x & -4y & +z & = & 6 \\ -3x & +6y & +5z & = & -1 \\ x & -3y & +7z & = & 0 \end{array}$$

11. Find the inverse of $\begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$.

*12. $n(S)=80$, $n(A)=40$, $n(B)=24$, $n(C)=28$, $n(A\cap B)=8$, $n(A\cap C)=11$, $n(B\cap C)=12$, and $n(A\cap B\cap C)=5$. Find $n(A\cap B')$.

13. An executive hires 3 office workers from 6 applicants. In how many ways can the group of 3 hires be selected?

14. An executive hires a secretary, a receptionist, and a typist from 6 applicants. In how many ways can the selection be made?

15. A trucking company is hiring 3 drivers and 4 clerks. There are 6 applicants for driver and 10 for clerk. In how many ways can the hiring be done?

16. In a group of 35 children, 10 have blonde hair, 14 have brown eyes, and 4 have both blonde hair and brown eyes. What is the probability that a child selected at random has blonde hair but does not have brown eyes?

17. An inspection at a furniture plant reveals that 2% of the desks have structural defects, 3% have finish defects, and 1% have both. Find the probability that a desk that is selected at random has no defects.

18. A writing class has 32 students: 16 are seniors, 12 are juniors, and 4 are sophomores. 9 of the seniors, 5 of the juniors, and 2 of the sophomores are journalism majors. What is the probability that a randomly selected student is a journalism major given that the student is a junior?

19. 100 students took a mathematics test: 15 received a grade of A, 20 had SAT mathematics scores above 550, and 10 both received a grade of A and had an SAT score above 550. Are the events "received a grade of A" and "SAT mathematics scores above 550" independent events?

20. The soft drinks in an ice chest are 55% Pepsi and 45% Coke. The Pepsi drinks are 40% diet, and the Coke drinks are 30% diet. A drink is randomly selected and is found to be diet. What is the probability that it is Coke?