

MAT 410 – 001 Fall 2008

Final Exam

Turn in no later than noon on Friday, December 19.

Do 10 problems.

1. G is a cyclic group. G has exactly 3 subgroups: G itself, $\{e\}$, and a subgroup of order 7. What is $|G|$?
2. Prove that an abelian group of order 6 is cyclic.
3. Prove that if x is the only element of order 2 in a group, then x lies in the center of the group.
4. $H \leq G$. Let $x \in G$. Show that $xHx^{-1} \leq G$.
5. Show that S_4 is not isomorphic to D_{24} .
6. $|G| = 25$. Show that either G is cyclic or $g^5 = e$ for all g in G .
7. $|G| = pq$. Where p and q are distinct primes. Prove that every subgroup of G is cyclic.
8. $|G| < \infty$. $H \trianglelefteq G$. Prove that the order of the element gH in G/H must divide the order of g in G .
9. $|G:H| = 2$. Prove that $H \trianglelefteq G$.
10. $\phi: \mathbb{Z}_{30} \rightarrow \mathbb{Z}_5$ is a homomorphism and is onto. What is the $\ker \phi$?
11. How many homomorphisms are there from \mathbb{Z}_{20} onto \mathbb{Z}_{10} ?
12. G is a finite abelian group. G has exactly one subgroup for each of the divisors of $|G|$. Show that G is cyclic.
13. $G = Q_8$. $H = \{1, -1\}$. Show that $H \trianglelefteq G$ and construct the Cayley table for G/H .