

MAT 410 – 001

Fall 2008

Do 8 problems.

The test is due on Thursday, December 4

1. Find subgroups H and K of D_8 such that $H \triangleleft K$ and $K \triangleleft D_8$ but H is not normal in D_8 .
2. $H \triangleleft G$ and $K \triangleleft G$. Show that $H \cap K \triangleleft G$.
3. $|a| = 24$. Find all left cosets of $\langle a^3 \rangle$ in $\langle a \rangle$.
4. For each subgroup of D_8 , determine the left cosets.
5. Prove that a quotient group of an abelian group is abelian.
6. Show that $\text{SL}(2, \mathbb{R}) \triangleleft \text{GL}(2, \mathbb{R})$.
7. Prove that every subgroup of Q_8 is normal.
8. Show that $\langle r^2, s \rangle$ in D_8 is isomorphic to V_4 .
9. G is an abelian group. $|G| = 120$. G has exactly three elements of order 2. What group is G ?
10. Show that $\mathbb{Z} \times \mathbb{Z} / (\langle (a, 0) \rangle \times \langle (0, b) \rangle) \cong \mathbb{Z}_a \times \mathbb{Z}_b$.