MAT 410 – 001
Fall 2008
Test Two

Do 10 problems.

The test is due on Thursday, November 13

1. D_8 has 7 cyclic subgroups. List them. Find a subgroup of D_8 of order 4 that is not cyclic.

2. $G = \langle a \rangle$ and $|a| = 24$. List all generators for the subgroup of order 8.

3. G is an abelian group. Let $H = \{g \in G \mid |g| \text{divides } 12\}$. Show that $H \leq G$.

4. p and q are distinct primes.

4a. Determine the subgroup lattice for $\mathbb{Z}_{p^2q}$.

4b. Determine the subgroup lattice for $\mathbb{Z}_{p^n}$ where n is a positive integer.

5. p is a prime. If a group has more than $p - 1$ elements of order p, why can't the group by cyclic?

6. Let p be a prime and let G be an abelian group. Show that the set of all elements whose orders are powers of p is a subgroup of G.

7. G is a group. $a \in G$. Show that $C(a) = C(a^{-1})$.

8. G is a group with exactly 8 elements of order 10. How many cyclic subgroups of order 10 does G have?

9. Determine each of the following: $Z(S_3)$, $Z(D_8)$, and $Z(Q_8)$.

10. Prove that $Z_2 \times Z_2$ is not cyclic.

11. $G = D_8$. $A = \{1, s, r^2, sr^2\}$. Show that $N_G(A) = G$.

12. Show that the mapping $\varphi : \mathbb{C} \to \mathbb{C}$ by $\varphi(a+bi) = a - bi$ is an automorphism of $\mathbb{C}$ under +.