MAT 410 – 001
Fall 2008
Test One

Do 10 problems.

The test is due on Wednesday, October 22.

1. G is a group. Show that G is abelian if and only if for all elements $a,b \in G$, $(ab)^{-1} = a^{-1}b^{-1}$.

2. G is a group. $a,b \in G$, and n is an integer. Show that $(a^{-1}b)^n a = a^{-1}b^n a$.

3. Let x belong to a group. If $x^2 \neq e$ and $x^6 = e$, prove that $x^4 \neq e$ and $x^5 \neq e$. What can we say about the order of x?

4. G is a finite group. Show that the number of elements $g \in G$ such that $g^3 = e$ is odd.

5. Consider the following Cayley table:

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	8	7	6	5	4	3
3	3	4	5	6	7	8	1	2
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	4	3	2	1	8	7
7	7	8	1	2	3	4	5	6
8	8	7	6	5	4	3	2	1

Find the order of each element. How are the orders of each element related to the order of the group?

6. In a group, if $|a| = n$ and k divides n, prove that $\left|a^{n/k}\right| = k$.

7. Complete the Cayley table:

	e	a	b	c	d
e	e	_	_	_	_
a	_	b	_	_	e
b	_	c	d	e	_
c	_	d	_	a	b
d	_	_	_	_	_

8. Determine the order of each of the elements of D_{30}.

9. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$.

9a. Write each as a product of disjoint cycles.

9b. Find the order of each.

9c. Compute α^{-1}.

9d. Compute $\alpha\beta$.

9e. Compute $\beta\alpha$.

10. In S_3 (which also may be thought of as D_6), find elements α and β so that $|\alpha| = 2, |\beta| = 2, |\alpha\beta| = 3$.

11. $G = \left\{ a + b\sqrt{2} \mid a,b \text{ are rational numbers} \right\}$ and $H = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a,b \text{ are rational numbers} \right\}$. Show that $G \cong H$.

12. G is a group. $a \in G$. Show that $\phi : G \to G$ defined by $\phi(g) = aga^{-1}$ is an isomorphism of G onto itself. (Such a mapping is called an automorphism of G.)