#1 Let x = number of acres of soybeans
Let y = number of acres of corn
Let z = number of acres of wheat

$$x + y + z = 500$$
$$20x + 30y + 40z = 14600$$
$$y = 2z$$

#2.

$$\begin{matrix} \\ R_2 - 3R_1 \\ R_3 + R_1 \end{matrix}\begin{bmatrix} \textcircled{1} & 2 & 1 & 2 \\ 3 & 6 & 4 & 7 \\ -1 & 1 & 5 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 6 & 12 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{3}R_2 \\ \\ \\ \end{matrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 6 & 12 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} R_1 - 2R_2 \\ \\ \\ \end{matrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & \textcircled{1} & 2 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{matrix} R_1 + 3R_3 \\ R_2 - 2R_3 \\ \\ \end{matrix}\begin{bmatrix} 1 & 0 & -3 & -6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \textcircled{1} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$x = -3$$
$$y = 2$$
$$z = 1$$

unique solution

#3a $x + z = -1$

$y - 2z = 3$.

#3b infinitely many solutions

$x = -1 - z$

$y = 3 + 2z$

#4

	x	y	s	t	u	p	
s	-2	1	1	0	0	0	4
t	1	-3	0	1	0	0	6
u	1	1	0	0	1	0	10
	-5	-3	0	0	0	1	0

4. Write the initial tableau for the following linear programming problem. You need not solve the problem.

Maximize $p = 5x + 3y$.

subject to the following constraints.

$$\begin{array}{rrcl} -2x & +y & \leq & 4 \\ x & -3y & \leq & 6 \\ x & +y & \leq & 10 \\ x & & \geq & 0 \\ & y & \geq & 0 \end{array}$$

5. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	2	2	1	0	0	0	28
t	3	1	1	0	1	0	0	24
u	1	4	0	0	0	1	0	24
	−6	−12	−8	0	0	0	1	0

test quotients
$28/2 = 14$
$24/1 = 24$
$24/4 = 6$ ←

Phase II

6. The following is an initial tableau. Determine the pivot.

		x	y	z	s	t	u	p	
*	s	2	2	−1	−1	0	0	0	18
*	t	2	1	−1	0	−1	0	0	15
*	u	0	2	−1	0	0	−1	0	24
		24	36	−20	0	0	0	1	0

test quotient
$18/2 = 9$
$15/2$ ←
$24/0$

Phase I

7. The following is a final tableau. Determine the maximum value of p and the values of x, y, and z.

	x	y	z	s	t	u	p	
t	100	0	0	200	2	−300	0	1000
z	0	0	200	−200	0	200	0	3000
y	100	100	0	200	0	−100	0	3000
	200	0	0	200	0	200	200	21000

$P = 21000/200$

$x = 0$

$y = 3000/100$

$z = 3000/200$

#8a

Maximize $P = 3x + 4y$

subject to

$$6x + 9y \leq 150$$
$$5x + 4y \leq 90$$
$$x \geq 0$$
$$y \geq 0$$

#8b

	x	y	s	t	P		
s	6	(9)	1	0	0	150	$150/9$ ←
t	5	4	0	1	0	90	$90/4$
	-3	-4	0	0	1	0	

↑

#8c

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of profit, x equal the number of units of product A, and y equal the number of units of product B. You need not solve the problem.

> A company manufactures two products A and B, on two machines, I and II. It has been determined that the company will realize a profit of \$3 per unit of product A and a profit of \$4 per unit of product B. To manufacture 1 unit of product A requires 6 minutes on machine I and 5 minutes on machine II. To manufacture 1 unit of product B requires 9 minutes on machine I and 4 minutes on machine II. There are 150 minutes of machine time available on machine I and 90 minutes of machine time available on machine II. How many units of each product should be manufactures in order to maximize profit?

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. The following is neither an initial nor a final tableau.

		x	y	z	s	t	u	p	
	z	5	10	15	−1	0	0	0	150
*	t	20	10	0	2	−3	0	0	300
*	u	35	10	0	2	0	−3	0	150
		4	2	0	1	0	0	3	−150

test quotients
$150/5 = 30$
$300/20 = 15$
$150/35 = 30/7$ ←

9a. Determine the values of x, y, z, s, t, u, and p at this stage.
$x=0,\ y=0,\ z=150/15,\ s=0,\ t=300/-3,\ u=150/-3,\ p=-150/3$

9b. Is the tableau in phase I or phase II? Phase I

9c. Determine the next pivot. You need not do the pivot operation.

#10 $$\begin{bmatrix} 3 & -5 & 4 \\ 4 & 2 & -3 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -12 \\ -2 \end{bmatrix}$$

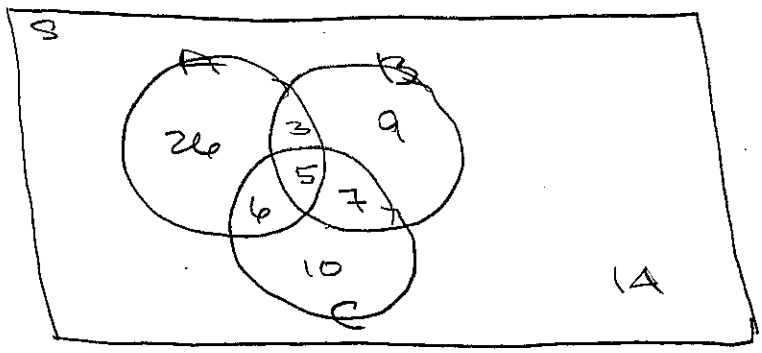
#11 $$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ -1 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 3 & -2 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -3 & 2 & -1 \end{array}\right]$$

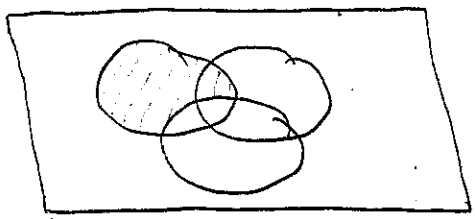
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 5 & -3 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & -3 & 2 & -1 \end{array}\right]$$

I $\qquad$ A^{-1}

#12



$A \cap C'$



$n(A \cap C') = 26 + 3 = 29$

#13 $C(7,4) = 35$

#14 $P(7,4) = 840$

#15 $C(7,2)\,C(9,2)\,C(5,2)$

$= 7560$

#16

$P(\text{blonde} \cup \text{brown})$

$= P(\text{blonde}) + P(\text{brown}) - P(\text{both})$

$= {}^{10}/_{35} + {}^{14}/_{35} - {}^{4}/_{35}$

$= {}^{20}/_{35} \approx 0.571$

#17

$P(\text{structural} \cup \text{guard})$

$= P(\text{structural}) + P(\text{guard}) - P(\text{both})$

$= 0.07 + 0.03 - 0.01$

$= 0.04$

#18

$$P(\text{graduate} \mid \text{senior}) = \frac{P(\text{graduate} \cap \text{senior})}{P(\text{senior})}$$

$$= \frac{9}{16} \approx 0.5625$$

#19

$P(\text{head}) = 8/16 = 1/2$

$P(\text{graduate}) = 4/16 = 1/4$

$P(\text{both}) = 2/16 = 1/8$

$1/2 * 1/4 \stackrel{?}{=} 1/8$

Yes.

independent events

#20 Bayes theorem

$$P(\gamma|A) = \frac{P(A|\gamma)P(\gamma)}{P(A|\gamma)P(\gamma) + P(A|so)P(so) + P(A|m)P(m)}$$

$$= \frac{5/25 * 25/50}{5/25 * 25/50 + 3/10 * 10/50 + 6/15 * 15/50}$$

$$= 5/14 \approx 0,3571$$