MAT 114 – 007
Fall 2008
Comprehensive Exam
Show all work.

1. Set up a system of linear equations to solve the following problem. You need not solve the system. Define the unknowns that you use as variables.

> A farmer wants to plant his entire 500 acre farm with a combination of soybeans, corn, and wheat. Each acre of soybeans requires 20 hours of labor per planting; each acre of corn, 30 hours; and each acre of wheat, 40 hours. He has 14,600 hours of labor available per planting season. If he wants to use all the labor available to him, and he wants to plant twice as many acres in corn as in wheat, how many acres of each crop should he plant?

*2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system. Determine whether there is a unique solution, no solution, or infinitely many solutions. If the solution is unique, state the values of x, y, and z. If there are infinitely many solutions, solve for the pivot variables in terms of the free variables.

$$\begin{aligned} x &+2y &+z &= 2 \\ 3x &+6y &+4z &= 7 \\ -x &+y &+5z &= 10 \end{aligned}$$

3. The augmented matrix of a system of linear equations (with unknowns x, y, z, and u) reduces to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

3a. Write the system of equations that corresponds to this augmented matrix.

3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4. Write the initial tableau for the following linear programming problem. You need not solve the problem.

Maximize $p = 5x + 3y$.

subject to the following constraints.

$$\begin{array}{rrcl} -2x & +y & \le & 4 \\ x & -3y & \le & 6 \\ x & +y & \le & 10 \\ x & & \ge & 0 \\ & y & \ge & 0 \end{array}$$

5. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	2	2	1	0	0	0	28
t	3	1	1	0	1	0	0	24
u	1	4	0	0	0	1	0	24
	–6	–1	–28	0	0	0	1	0

6. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	2	2	−1	−1	0	0	0	18
t	2	1	−1	0	−1	0	0	15
u	0	2	−1	0	0	−1	0	224
	24	36	−20	0	0	0	1	0

7. The following is a final tableau. Determine the maximum value of p and the values of x, y, and z.

	x	y	z	s	t	u	p	
t	100	0	0	200	2	−300	0	1000
z	0	0	200	−200	0	200	0	3000
y	100	100	0	200	0	−100	0	3000
	200	0	0	200	0	200	200	21000

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of profit, x equal the number of units of product A, and y equal the number of units of product B. You need not solve the problem.

> A company manufactures two products A and B, on two machines, I and II. It has been determined that the company will realize a profit of \$3 per unit of product A and a profit of \$4 per unit of product B. To manufacture 1 unit of product A requires 6 minutes on machine I and 5 minutes on machine II. To manufacture 1 unit of product B requires 9 minutes on machine I and 4 minutes on machine II. There are 150 minutes of machine time available on machine I and 90 minutes of machine time available on machine II. How many units of each product should be manufactures in order to maximize profit?

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. The following is neither an initial nor a final tableau.

	x	y	z	s	t	u	p	
z	5	10	15	–1	0	0	0	150
t	20	10	0	2	–3	0	0	300
u	35	10	0	2	0	–3	0	150
	4	2	0	1	0	0	3	–150

9a. Determine the values of x, y, z, s, t, u, and p at this stage.

9b. Is the tableau in phase I or phase II?

9c. Determine the next pivot. You need not do the pivot operation.

10. Translate the given system of linear equations into matrix form.

$$\begin{aligned} 3x &- 5y &+ 4z &= 10 \\ 4x &+ 2y &- 3z &= -1 \\ -x & & + z &= 2 \end{aligned}$$

11. Find the inverse of $\begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -1 \\ -1 & 1 & 3 \end{bmatrix}$.

*12. $n(S)=80$, $n(A)=40$, $n(B)=24$, $n(C)=28$, $n(A\cap B)=8$, $n(A\cap C)=11$, $n(B\cap C)=12$, and $n(A\cap B\cap C)=5$. Find $n(A\cap C')$.

13. In how many ways can a committee of 4 be selected from a group of 7 people?

14. In how many ways can a slate of officers – chair, vice-chair, secretary, and treasurer be selected from a group of 7 people?

15. From a set of 7 mathematics books, 9 science books, and 5 literature books, in how many ways can a student select 2 of each kind?

16. In a group of 35 children, 10 have blonde hair, 14 have brown eyes, and 4 have both blonde hair and brown eyes. What is the probability that a child selected at random has blonde hair or brown eyes?

17. An inspection at a furniture plant reveals that 2% of the desks have structural defects, 3% have finish defects, and 1% have both. Find the probability that a desk that is selected at random has at least one type of defect.

18. A writing class has 32 students: 16 are seniors, 12 are juniors, and 4 are sophomores. 9 of the seniors, 5 of the juniors, and 2 of the sophomores are journalism majors. What is the probability that a randomly selected student is a journalism major given that the student is a senior?

19. A kennel raises purebred dogs. Several litters from one dog produced 16 puppies with the following markings: 8 had a white mark on the head, 4 had a white mark on the forelegs, 2 had a white mark on both head and forelegs, and 6 had neither mark. Are the events “white mark on the head” and white mark on the forelegs” independent event?

20. A class is composed of 10 sophomores, 25 juniors, and 15 seniors. On the first test, 3 sophomores, 5 juniors, and 6 seniors earned As. Given that an randomly selected student received an A, what is the probability that the student is a junior?