#1 Let $x$ = the number of children
$y$ = the number of students
$z$ = the number of adults

$$x + y + z = 900$$
$$25x + 3y + 4z = 2800$$

$$z = \tfrac{1}{2}(x + y)$$
$$2z = x + y$$
$$x + y - 2z = 0$$

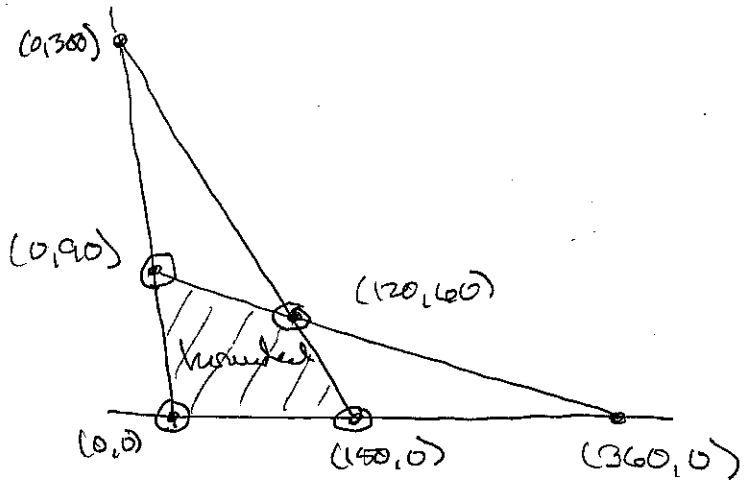
#2
$$\begin{matrix} \\ 2R_2 - R_1 \\ R_3 - R_1 \end{matrix}\begin{bmatrix} \textcircled{2} & 1 & 1 & 180 \\ 1^{2} & 3^{6} & 2^{4} & 300^{600} \\ 2 & 1 & 2 & 240 \end{bmatrix} \xrightarrow{5R_1 - R_2} \begin{bmatrix} 2^{10} & 1 & 1 & 180^{900} \\ 0 & \textcircled{5} & 3 & 420 \\ 0 & 0 & 1 & 60 \end{bmatrix}$$

$$\begin{matrix} R_1 - 2R_3 \\ R_2 - 3R_3 \end{matrix}\begin{bmatrix} 10 & 0 & 2 & 480 \\ 0 & 5 & 3 & 420 \\ 0 & 0 & \textcircled{1} & 60 \end{bmatrix} \xrightarrow[\frac{1}{5}R_2]{\frac{1}{10}R_1} \begin{bmatrix} 10 & 0 & 0 & 360 \\ 0 & 5 & 0 & 240 \\ 0 & 0 & 1 & 60 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 36 \\ 0 & 1 & 0 & 48 \\ 0 & 0 & 1 & 60 \end{bmatrix}$$

$x = 36$
$y = 48$
$z = 60$

#3a




| Corners | $P=10x+15y$ | |
|---|---|---|
| (0,90) | 1350 | |
| (0,0) | 0 | |
| (120,60) | 2100 | Maximum |
| (150,0) | 1500 | |

#4

| | $x$ | $y$ | $z$ | $s$ | $t$ | $P$ | constants | |
|---|---|---|---|---|---|---|---|---|
| $s$ | 2 | 1 | 2 | 1 | 0 | 0 | 13 | $13/1$ |
| $t$ | 1 | (1) | −3 | 0 | 1 | 0 | 8 | $8/1$ ← |
| $P$ | −2 | −3 | −2 | 0 | 0 | 1 | 0 | |

↑

#5 Minimize $C = 30x + 40y + 50z$

$\Leftrightarrow$

Maximize $P = -30x - 40y - 50z$

| | x | y | z | s | t | P | constants | |
|---|---|---|---|---|---|---|---|---|
| * s | 10 | (14) | 5 | −1 | 0 | 0 | 220 | $\frac{220}{14}$ ← |
| * t | 5 | 3 | 9 | 0 | −1 | 0 | 340 | $\frac{340}{3}$ |
| P | 30 | 40 | 50 | 0 | 0 | 1 | 0 | |

column

#6

| | x | y | z | s | t | u | P | constants | |
|---|---|---|---|---|---|---|---|---|---|
| s | 7 | 12 | 12 | 1 | 0 | 0 | 0 | 312 | $\frac{312}{12}$ |
| * t | 13 | (20) | 12 | 0 | −1 | 0 | 0 | 384 | $\frac{384}{20}$ ← |
| * u | 5 | 4 | 12 | 0 | 0 | −1 | 0 | 192 | $\frac{192}{4}$ |
| P | −14 | −24 | −26 | 0 | 0 | 0 | 1 | 0 | |

#7

Maximize $P = 8x + 6y + 9z$

subject to

$$x + y + z \leq 1900$$
$$6x + 5y + 8z \leq 13400$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

#8

| | | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ | $P$ | constants | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $s$ | 6 | 1 | 5 | 1 | 0 | 0 | 0 | 68 | 68/6 |
| * | $t$ | (4) | 3 | 1 | 0 | −1 | 0 | 0 | 32 | 32/4 ← |
| * | $u$ | 2 | 4 | 3 | 0 | 0 | −1 | 0 | 36 | 36/2 |
| | | −8 | −2 | −6 | 0 | 0 | 0 | 1 | 0 | |

Phase I

#9

| | x | y | z | s | t | u | p | Constants | |
|---|---|---|---|---|---|---|---|---|---|
| s | 5 | (4) | 3 | 1 | 0 | 0 | 0 | 8 | 8/4 |
| t | 2 | 7 | 1 | 0 | 1 | 0 | 0 | 15 | 15/7 |
| u | 6 | 8 | 5 | 0 | 0 | 1 | 0 | 24 | 24/8 |
| P | -8 | -10 | -4 | 0 | 0 | 0 | 1 | 0 | |

Phase II

#10

| | x | y | z | t | u | v | p | Constants | |
|---|---|---|---|---|---|---|---|---|---|
| s | 0 | -1 | 1 | 0 | 0 | -1 | 0 | 10 | 10/1 |
| t | 0 | 2 | 0 | 1 | 0 | -1 | 0 | 35 | 35/2 |
| * u | 0 | (1) | 0 | 0 | -1 | -2 | 0 | 10 | 10/1 |
| x | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 25 | 25/0 |
| P | 0 | -1 | 0 | 0 | 0 | 2 | 1 | 50 | |

Phase I

$x = 25/1$

$y = 0$

$s = 10/1$

$t = 35/1$

$u = 10/-1$

$v = 0$

$P = 50/1$

#11

$x = 450/3$

$y = 9000/4$

$P = 10050/4$