

MAT 114 –007

Fall 2009

Test One

Show all work.

1. A theater has seating capacity for 900 and charges \$2 for children, \$3 for students, and \$4 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The total receipts were \$2800. Write a system of linear equations that determines the number of children, the number of students, and the number of adults who were at the screening. You need not solve the system.

2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$2x + y + z = 180$$

$$x + 3y + 2z = 300$$

$$2x + y + 2z = 240$$

3a. Graph the feasible set of the following set of linear inequalities.

$$\begin{aligned}x + 4y &\leq 360 \\2x + y &\leq 300 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

3b. Find the corner points of the feasible set.

3c. Is the feasible set bounded?

3d. By evaluating  $p = 10x + 15y$  at the corners determine the maximum value of  $p$  that satisfies the constraints.

4a. Write the initial tableau for the following standard maximum problem. You need not solve the problem.

$$\text{Maximize } p = 2x + 3y + 2z$$

subject to the following structural constraints and nonnegativity.

$$\begin{aligned}2x + y + 2z &\leq 13 \\x + y - 3z &\leq 8\end{aligned}$$

4b. Determine the first pivot. You do not need to do the pivot – just determine the row and column of the first pivot.

5a. Write the initial tableau for the following minimum problem. You need not solve the problem.

$$\text{Minimize } c = 30x + 40y + 50z$$

subject to the following structural constraints and nonnegativity.

$$\begin{array}{rclcl} 10x & +14y & +5z & \geq & 220 \\ 5x & +3y & +9z & \geq & 340 \end{array}$$

5b. Determine the first pivot. You do not need to do the pivot – just determine the row and column of the first pivot.

6a. Write the initial tableau for the following maximum problem. You need not solve the problem.

$$\text{Maximize } p = 14x + 24y + 26z$$

subject to the following structural constraints and nonnegativity.

$$\begin{array}{rclcl} 7x & +12y & +12z & \leq & 312 \\ 13x & +20y & +12z & \geq & 384 \\ 5x & +4y & +12z & \geq & 192 \end{array}$$

6b. Determine the first pivot. You do not need to do the pivot – just determine the row and the column of the first pivot.

7. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let  $p$  equal profit,  $x$  equal the number of barrels of gasoline,  $y$  equal the number of barrels of diesel fuel, and  $z$  equal the number of barrels of heating oil that are produced. You need not solve the problem.

The maximum daily production of an oil refinery is 1900 barrels. The refinery can produce three types of fuel: gasoline, diesel fuel, and heating oil. The production cost per barrel is \$6 of gasoline, \$5 for diesel fuel, and \$8 for heating oil. The daily production budget is \$13400. The profit is \$8 per barrel for gasoline, \$6 per barrel for diesel fuel, and \$9 per barrel for heating oil. How much of each should be produced to maximize profit?

8. The following is an initial tableau.

|     | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ | $p$ | constants |
|-----|-----|-----|-----|-----|-----|-----|-----|-----------|
| $s$ | 6   | 1   | 5   | 1   | 0   | 0   | 0   | 68        |
| $t$ | 4   | 3   | 1   | 0   | -1  | 0   | 0   | 32        |
| $u$ | 2   | 4   | 3   | 0   | 0   | -1  | 0   | 36        |
| $p$ | -8  | -2  | -6  | 0   | 0   | 0   | 1   | 0         |

8a. Is this tableau in phase I or phase II?

8b. Determine the pivot.

9. The following is an initial tableau.

|     | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ | $p$ | constants |
|-----|-----|-----|-----|-----|-----|-----|-----|-----------|
| $s$ | 5   | 4   | 3   | 1   | 0   | 0   | 0   | 8         |
| $t$ | 2   | 7   | 1   | 0   | 1   | 0   | 0   | 15        |
| $u$ | 6   | 8   | 5   | 0   | 0   | 1   | 0   | 24        |
| $p$ | -8  | -1  | 0-4 | 0   | 0   | 0   | 1   | 0         |

9a. Is this tableau in phase I or phase II?

9b. Determine the pivot.

10. The following tableau is neither an initial tableau nor a final tableau.

|     | $x$ | $y$ | $s$ | $t$ | $u$ | $v$ | $p$ | constants |
|-----|-----|-----|-----|-----|-----|-----|-----|-----------|
| $s$ | 0   | -1  | 1   | 0   | 0   | -1  | 0   | 10        |
| $t$ | 0   | 2   | 0   | 1   | 0   | -1  | 0   | 35        |
| $u$ | 0   | 1   | 0   | 0   | -1  | -2  | 0   | 1 0       |
| $x$ | 1   | 0   | 0   | 0   | 0   | 1   | 0   | 2 5       |
| $p$ | 0   | -1  | 0   | 0   | 0   | 2   | 1   | 50        |

10a. Is this tableau in phase I or phase II?

10b. Determine the pivot.

10c. Determine the values of  $x$ ,  $y$ ,  $s$ ,  $t$ ,  $u$ ,  $v$ , and  $p$  at this stage.

11. The following is a final tableau. Determine the maximum value of  $p$  and the values of  $x$  and  $y$ . (You may show the values of  $x$ ,  $y$ , and  $p$  as fractions.)

|     | $x$ | $y$ | $s$ | $t$ | $u$ | $p$ | constants |
|-----|-----|-----|-----|-----|-----|-----|-----------|
| $x$ | 3   | 0   | 0   | -1  | 1   | 0   | 900       |
| $s$ | 0   | 0   | 3   | 1   | -1  | 0   | 900       |
| $y$ | 0   | 9   | 0   | 5   | -2  | 0   | 9000      |
| $p$ | 0   | 0   | 0   | 5   | 16  | 4   | 10000     |