

MAT 114 – 001
Fall 2008
Comprehensive Exam
Show all work.

1. Set up a system of linear equations to solve the following problem. You need not solve the system. Define the variables that you use.

The Dandy Donut Company makes three types of donuts: cake, glazed, and jelly. Each dozen of cake donuts requires 2 eggs, 2 cups of flour, and 1 cup of sugar. Each dozen glazed donuts requires 1 egg, 2 cups of flour, and 1 cup of sugar. Each dozen jelly donuts requires 3 eggs, 3 cups of flour, and 2 cups of sugar. Suppose that 100 eggs, 130 cups of flour, and 70 cups of sugar were used on a given day. How many donuts of each type were produced?

- *2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system. Determine whether there is a unique solution, no solution, or infinitely many solutions. If the solution is unique, state the values of x , y , and z . If there are infinitely many solutions, solve for the pivot variables in terms of the free variables.

$$\begin{array}{rclcl} x & +2y & -3z & = & 5 \\ 2x & +3y & -4z & = & 7 \\ -3x & -4y & +5z & = & -9 \end{array}$$

3. The augmented matrix of a system of linear equations (with unknowns x , y , z , and u) reduces to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 4 \end{array} \right]$$

- 3a. Write the system of equations that corresponds to this augmented matrix.
- 3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4. Write the initial tableau for the following linear programming problem. You need not solve the problem.

$$\text{Maximize } p = 3x - y.$$

subject to the following constraints.

$$x + 2y \leq 80$$

$$-x + 3y \leq 60$$

$$x \leq 50$$

$$x \geq 0$$

$$y \geq 0$$

5. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	2	2	1	0	0	0	30
t	3	0	1	0	1	0	0	36
u	0	2	1	0	0	1	0	14
	-4	-8	-6	0	0	0	1	0

6. The following is an initial tableau. Determine the pivot.

	x	y	z	s	t	u	p	
s	4	1	-1	-1	0	0	0	6
t	3	1	0	0	-1	0	0	5
u	2	1	0	0	0	-1	0	4
	36	12	-7	0	0	0	1	0

7. The following is a final tableau. Determine the maximum value of p and the values of x , y , and z .

	x	y	z	s	t	u	p	
y	0	6	0	1	-1	3	0	1200
x	4	0	0	1	1	-3	0	400
z	0	0	6	-1	1	3	0	900
	0	0	0	5	1	9	12	7800

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of profit, x equal the number of machine A, and y equal the number of machine B produced. You need not solve the problem.

National Business Machines Corporation manufactures two models of fax machines: A and B. Each model A costs \$100 to make, and each model B costs \$150. The profits are \$30 for each model A and \$40 for each model B fax machine. If the total number of fax machines demanded each month does not exceed 2500 and the company has earmarked no more than \$600,000 per month for manufacturing costs, find how many units of each model the company should make each month to maximize its monthly profits.

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. The following is neither an initial nor a final tableau.

	x	y	z	s	t	u	p	
s	2	0	12	4	0	-1	0	2000
t	0	0	-8	0	1	-2	0	2000
y	2	4	8	0	0	1	0	4000
	-2	0	13	0	0	2	1	8000

9a. Determine the values of x , y , z , s , t , u , and p at this stage.

9b. Is the tableau in phase I or phase II?

9c. Determine the next pivot. You need not do the pivot operation.

10. Translate the given system of linear equations into matrix form.

$$\begin{array}{rclcrcl} 2x & -4y & +z & = & 6 \\ -3x & +6y & +5z & = & -1 \\ x & -3y & +7z & = & 0 \end{array}$$

11. Find the inverse of $\begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$.

*12. $n(S)=80$, $n(A)=40$, $n(B)=24$, $n(C)=28$, $n(A \cap B)=8$,
 $n(A \cap C)=11$, $n(B \cap C)=12$, and $n(A \cap B \cap C)=5$. Find $n(A \cap B')$.

13. An executive hires 3 office workers from 6 applicants. In how many ways can the group of 3 hires be selected?

14. An executive hires a secretary, a receptionist, and a typist from 6 applicants. In how many ways can the selection be made?

15. A trucking company is hiring 3 drivers and 4 clerks. There are 6 applicants for driver and 10 for clerk. In how many ways can the hiring be done?

16. In a group of 35 children, 10 have blonde hair, 14 have brown eyes, and 4 have both blonde hair and brown eyes. What is the probability that a child selected at random has blonde hair but does not have brown eyes?

17. An inspection at a furniture plant reveals that 2% of the desks have structural defects, 3% have finish defects, and 1% have both. Find the probability that a desk that is selected at random has no defects.
18. A writing class has 32 students: 16 are seniors, 12 are juniors, and 4 are sophomores. 9 of the seniors, 5 of the juniors, and 2 of the sophomores are journalism majors. What is the probability that a randomly selected student is a journalism major given that the student is a junior?
19. 100 students took a mathematics test: 15 received a grade of A, 20 had SAT mathematics scores above 550, and 10 both received a grade of A and had an SAT score above 550. Are the events “received a grade of A” and “SAT mathematics scores above 550” independent events?
20. The soft drinks in an ice chest are 55% Pepsi and 45% Coke. The Pepsi drinks are 40% diet, and the Coke drinks are 30% diet. A drink is randomly selected and is found to be diet. What is the probability that it is Coke?