

MAT 114 – 001
Fall 2008
Review for Test One
Show all work.

1. A farmer wants to plant his entire 500 acre farm with a combination of soybeans, corn, and wheat. Each acre of soybeans requires 20 hours of labor per planting season; each acre of corn, 30 hours; and each acre of wheat, 40 hours. He has 14600 hours of labor available each planting season. If he wants to use all the labor available to him, and he wants to plant twice as many acres in corn as in wheat, how many acres of each crop should he plant? Let x equal the number of acres of soybeans, y equal the number of acres of corn, and z equal the number of acres of wheat. Write a system of linear equation to determine x , y , and z . You need not solve the system.

2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$\begin{array}{rclcl} x & -2y & +3z & = & 3 \\ x & -y & +z & = & 5 \\ 2x & -2y & +4z & = & 12 \end{array}$$

3. The augmented matrix of a system of linear equations (with unknowns x , y , and z) reduces to

$$\left[\begin{array}{cccc} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 4 \end{array} \right]$$

3a. Write the system of equations that corresponds to this augmented matrix.

3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4a. Graph the feasible set of the following set of linear inequalities.

$$2x + 3y \leq 12$$

$$2x + y \leq 8$$

$$x \geq 0$$

$$y \geq 0$$

4b. Find the corner points of the feasible set.

4c. Is the feasible set bounded?

4d. By evaluating $p = 4x + 3y$ at the corners determine the maximum value of P that satisfies the constraints.

5. Write the initial tableau for the following standard maximum problem. You need not solve the problem.

$$\text{Maximize } p = 6x + 5y + 3z$$

subject to the following structural constraints and nonnegativity.

$$x + 2y + 3z \leq 6$$

$$2x - 10y + 2z \leq 10$$

$$4x + 8y - z \leq 32$$

6. The following is an initial tableau.

6a. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	1	5	1	0	0	0	1500
t	4	8	8	0	1	0	0	10000
u	2	4	8	0	0	1	0	4000
p	-6	-8	-3	0	0	0	1	0

6b. Do this one pivot operation.

7. The following is a final tableau. Determine the maximum value of p and the values of x , y , and z . (Show the values of x , y , z , and p as fractions.)

	x	y	z	s	t	u	p	
x	14	0	3	2	0	1	0	5000
t	-2	0	0	0	1	-1	0	500
y	-1	3	0	-1	0	1	0	500
p	71	0	0	8	0	10	3	32000

8. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal profit, x equal the number of tables, y equal the number of chairs, and z equal the number of bookcases that are produced. You need not solve the problem.

A furniture company produces tables, chairs, and bookcases. Each product must be assembled in the carpentry shop, sanded and smoothed in the finishing shop, and stained in the painting shop. The time for each of these activities is shown in the table below:

	Table	Chair	Bookcase
Carpentry	3	2	2.5
Finishing	1	1	1
Painting	2.5	1.5	2

Each table yields a profit of \$27, each chair a profit of \$15, and each bookcase a profit of \$24. Suppose that the company has 120 hours of carpentry time, 70 hours of finishing time, and 50 of painting time available each week. How many tables, chairs, and bookcases should be produced each week to maximize profits?