

Some stuff

Error bounds for trapezoidal, midpoint, and Simpson's rule:

$$\text{Error}(T_n) \leq \frac{K_2(b-a)^3}{12n^2}$$

$$\text{Error}(M_n) \leq \frac{K_2(b-a)^3}{24n^2}$$

$$\text{Error}(S_n) \leq \frac{K_4(b-a)^5}{180n^4}$$

Trig identities

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

An antiderivative

$$\int \sec u \, du = \ln |\sec u + \tan u| + c$$

Some Maclaurin Series

$$\frac{1}{1-u} = 1 + u + u^2 + u^3 + u^4 + u^5 + \dots = \sum_{k=0}^{\infty} u^k \quad -1 < u < 1$$

$$e^u = 1 + u + \frac{u^2}{2} + \frac{u^3}{3!} + \frac{u^4}{4!} + \frac{u^5}{5!} + \dots = \sum_{k=0}^{\infty} \frac{u^k}{k!} \quad -\infty < u < \infty$$

$$\sin u = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \frac{u^9}{9!} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{u^{2k+1}}{(2k+1)!} \quad -\infty < u < \infty$$

$$\cos u = 1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!} + \frac{u^8}{8!} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{u^{2k}}{(2k)!} \quad -\infty < u < \infty$$

$$\tan^{-1} u = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \frac{u^9}{9} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{u^{2k+1}}{2k+1} \quad -1 \leq u \leq 1$$

$$\ln(1-u) = -u - \frac{u^2}{2} - \frac{u^3}{3} - \frac{u^4}{4} - \dots = -\sum_{k=1}^{\infty} \frac{u^k}{k} \quad -1 < u < 1$$

$$\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} - \dots = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{u^k}{k} \quad -1 < u < 1 \text{ and } u = 1$$

Error bound for Taylor polynomial

$$|T_n(x) - f(x)| \leq K \frac{|x-a|^{n+1}}{(n+1)!}$$

MAT 222 - 061
Spring 2008 Comprehensive Exam
Show all work.

1. Integrate $\int_4^{\infty} \frac{1}{x(\ln x)^3} dx$. Show all steps.

Determine whether each of the following series (# 2 - 11) is **absolutely convergent**, **conditionally convergent**, or **diverges**. If the series is a convergent geometric series, find its sum. State the test you are using and show all details of the test. Always verify the conditions needed for the test to be valid.

2. $\sum_{k=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 5 \cdot 8 \cdots (3k-1)}$.

3. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2 + 2k}}$.

4. $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \cdots$.

5. $\sum_{k=1}^{\infty} \frac{e^{2k}}{k^k}$.

6. $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \cdots$.

7. $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k^3}}$.

8. $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k^3-1}$.

9. $\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt{k}}{k+1}$.

10. $\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+1}}$.

11. $\sum_{k=1}^{\infty} \frac{k+1}{k(k+2)}$.

12. Determine the interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{(k+1)(k+2)}.$$

13. Determine the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} (x-5)^k}{k 5^k}.$$

14. Find the Maclaurin series for $\frac{4}{2+3x}$. Where does the series converge?

15. Find a Maclaurin series for $\int_0^x (e^{-t^2} - 1) dt$.

16. Use a Maclaurin series to calculate $\int_0^{0.5} \cos \sqrt{x} dx$ so that the absolute value of the error is less than 0.0005.