$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2 2^k}{k!}$$

#1 $$\lim_{k\to\infty} \left| \frac{\frac{(-1)^{k+2}(k+1)^2 2^{k+1}}{(k+1)!}}{\frac{(-1)^{k+1} k^2 2^k}{k!}} \right|$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 \frac{2^{k+1}}{2^k} \frac{k!}{(k+1)!}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 2 \frac{1}{k+1} = 0$$

series converges by ratio test

#2 series of absolute values

$$\sum_{k=1}^{\infty} \frac{k}{k^2+1}$$

$$\lim_{k\to\infty} \frac{\frac{k}{k^2+1}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{k^2}{k^2+1} = 1$$

diverges by limit comparison to harmonic series NOT absolutely convergent

check for conditional convergence

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1} k}{k^2+1}$$

alternates

decreasing $$\left(\frac{k}{k^2+1}\right)' = -\frac{k^2-1}{(k^2+1)^2}$$

$$\lim_{k\to\infty} \frac{k}{k^2+1} = 0$$

converges conditionally by alternating series test

#3 $\sum_{k=1}^{\infty} \frac{1+k+k^2}{\sqrt{1+k^2+k^6}}$ looks like $\frac{k^2}{\sqrt{k^6}} = \frac{k^2}{k^3} = \frac{1}{k}$

$$\lim_{k\to\infty} \frac{\frac{1+k+k^2}{\sqrt{1+k^2+k^6}}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{k+k^2+k^3}{\sqrt{1+k^2+k^6}} = 1$$

series diverges by limit comparison to harmonic series

#4 $\sum_{k=1}^{\infty} (-1)^k \frac{k}{5+k}$

$$\lim_{k\to\infty} (-1)^k \frac{k}{5+k} \overset{?}{=} \pm 1$$

limit DNE

series diverges by the test for divergence

#5 $\sum_{k=1}^{\infty} \frac{k^2+1}{k^3-1}$

$\frac{k^2+1}{k^3-1} > \frac{k^2}{k^3} = \frac{1}{k}$

$\sum_{k=1}^{\infty} \frac{k^2+1}{k^3-1} > \sum_{k=1}^{\infty} \frac{1}{k}$ harmonic series diverges

series diverges by comparison to harmonic series

#6 $\sum_{k=3}^{\infty} \frac{1}{k \ln k \ln(\ln k)}$

$f(x) = \frac{1}{x \ln x \ln(\ln x)}$

is positive, continuous, and decreasing

$\int_3^{\infty} \frac{1}{x \ln x \ln(\ln x)} dx = \infty$

series diverges by the integral test

#7 $$\sum_{k=1}^{\infty} \frac{e^k}{3^{k-1}} = e + \frac{e^2}{3} + \frac{e^3}{9} + \frac{e^4}{27} + \cdots$$

geometric series
$r = e/3$ converges

$$\text{sum} = \frac{e}{1-e/3}$$

#8 $$\sum_{k=1}^{\infty} \frac{(-1)^k k}{4^k}$$

alternates

$$\left.\begin{array}{l} \frac{k}{4^k} > \frac{k+1}{4^{k+1}} \\ \lim_{k\to\infty} \frac{k}{4^k} = 0 \end{array}\right\}$$ converges by alternating series test

use n terms to approximate the sum
choose n so that

$$|n+1\text{st term}| = \left|\frac{(-1)^{n+1}(n+1)}{4^{n+1}}\right| < 0.0005$$

need $n \geq 6$

$$\sum_{k=1}^{6} \frac{(-1)^k k}{4^k} \approx -0.1596679687 5$$

#9 $\sum_{k=1}^{\infty} \frac{k(x-4)^k}{k^3+1}$ center is at 4

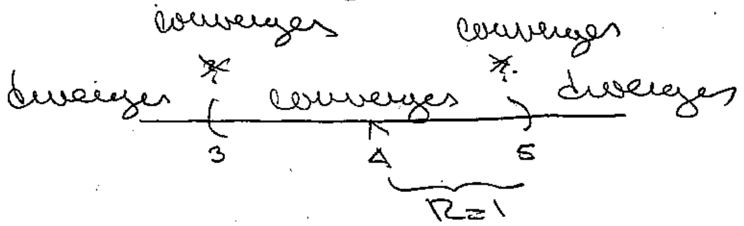
ratio test

$$\lim_{k\to\infty}\left|\frac{\frac{(k+1)(x-4)^{k+1}}{(k+1)^3+1}}{\frac{k(x-4)^k}{k^3+1}}\right| = \lim_{k\to\infty}\frac{k+1}{k}\,\frac{k^3+1}{(k+1)^3+1}\,|x-4|$$

$$= |x-4|$$

converges if

$$|x-4|<1$$
$$-1 < x-4 < 1$$
$$3 < x < 5$$



If $x=5$ $\quad \sum_{k=1}^{\infty} \frac{k}{k^3+1}$

$$\frac{k}{k^3+1} < \frac{k}{k^3} = \frac{1}{k^2}$$

$$\sum_{k=1}^{\infty} \frac{k}{k^3+1} < \sum_{k=1}^{\infty} \frac{1}{k^2}$$

p-series
p=2
converges

If $x=3$ $\quad \sum_{k=1}^{\infty} \frac{(-1)^k k}{k^3+1}$ is absolutely convergent

yes