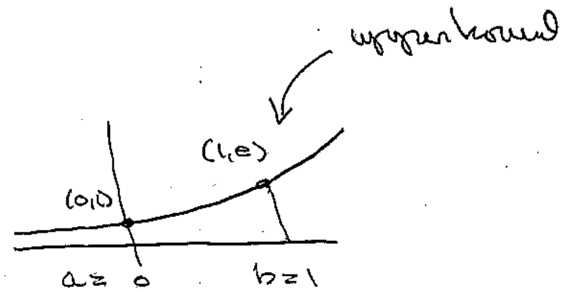
#1 $$\int_0^\infty \frac{x}{(x^2+1)^2}\,dx = \lim_{R\to\infty} \int_0^R \frac{x}{(x^2+1)^2}\,dx$$

$$= \lim_{R\to\infty} \left.\frac{-1}{2(x^2+1)}\right|_0^R = \lim_{R\to\infty} \frac{-1}{2(R^2+1)} + \frac{1}{2} = \frac{1}{2}$$

#2 $f(x) = e^x$

$f^{(4)}(x) = e^x$

$K_4 \le e$



$$\text{Error}(S_8) \le \frac{e(1-0)^5}{180(8)^4} \approx 0.000003687$$

#3 $n =$ 1 2 3 4 5

$a_n = \frac{3+5n^2}{n+n^2}$ 4 23/6 4 83/20 64/15

$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = 5$ converges

#4 $5 - 10/3 + 20/9 - 40/27 + \cdots$

$* -2/3$ $* -2/3$ $* -2/3$

$c = 5$

$r = -2/3.$

geometric series

$|r| = |-2/3| < 1$

converges to $\frac{5}{1-(-2/3)} = 3$

#5 $\sum_{k=1}^{\infty} \frac{k^2}{5k^2+4}$

$\lim_{k \to \infty} \frac{k^2}{5k^2+4} = 1/5 \neq 0$

series diverges by the test for divergence

#6 $\sum_{k=3}^{\infty} \frac{5}{k-2}$

$f(x) = \frac{5}{x-2}$ continuous, positive, decreasing

$\int_3^{\infty} \frac{5}{x-2}\,dx$ = diverges

The series diverges by the integral test.

#7 $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

$= \sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}} = \sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$ p-series $p = 3/2$ converges

#8 $\sum_{k=1}^{\infty} k e^{-x^2}$

$f(x) = x e^{-x^2}$ continuous, positive, decreasing

$\int_1^{\infty} x e^{-x^2} dx = \frac{1}{2e}$ converges

Series converges by the integral test.