MAT 222 - 061
Spring 2008
Test One
Show all work.

Error bounds

$$\text{Error}\left(T_n\right) \le \frac{K_2\left(b-a\right)^3}{12n^2}$$

$$\text{Error}\left(M_n\right) \le \frac{K_2\left(b-a\right)^3}{24n^2}$$

$$\text{Error}\left(S_n\right) \le \frac{K_4\left(b-a\right)^5}{180n^4}$$

1. $\int_0^\infty \frac{x}{\left(x^2+1\right)^2}\,dx$. Show the details of each step.

2. Estimate the error when Simpson's rule with $n = 8$ is used to approximate $\int_0^1 e^x\,dx$. You need not do the approximation.

3. Write the first five terms of the sequence

$$a_n = \frac{3+5n^2}{n+n^2}$$

Determine whether the sequence converges or diverges. If it converges, find its limit.

Determine whether each of the following series converges or diverges. If the series is a convergent geometric series, find its sum. State the test you are using and show all details of the test. Always verify the conditions needed for the test to be valid.

4. $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots.$

5. $\sum_{k=1}^{\infty}\frac{k^2}{5k^2+4}.$

6. $\sum_{k=3}^{\infty}\frac{5}{k-2}.$

7. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots.$

8. $\sum_{k=1}^{\infty}ke^{-k^2}.$