

MAT 221 – 051
Spring 2008
Review for Comprehensive Exam

Show all work.

On the integration problems, show the details of each step.

Some stuff:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

reference angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
	0°	30°	45°	60°	90°	180°	270°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-

1. Integrate $\int x^3 e^{x^2} dx$.

2a. Set up an integral that determines the surface area generated by revolving the portion of the curve $y = \frac{1}{x^2}$ from $\left(-4, \frac{1}{16}\right)$ to $(-1, 1)$ about the x -axis.

2b. Set up an integral that determines the surface area generated by revolving the portion of the curve $y = \frac{1}{x^2}$ from $\left(-4, \frac{1}{16}\right)$ to $(-1, 1)$ about the y -axis.

4. In a disastrous flight, an experimental paper airplane follows the path

$$\begin{aligned}x &= t - 3\sin t \\y &= 4 - 3\cos t\end{aligned} \quad t \geq 0$$

but crashes into a wall when $t = 10$.

3a. At what times was the plane flying horizontally?

3b. At what times was the plane flying vertically?

If you want to see the flight, graph the parametric curve in the window

$$\begin{aligned}0, 10, .05 \\-3, 12, 1 \\0, 8, 1\end{aligned}$$

4. How far did the plane in exercise 3 fly? I.e., what is the arc length of the parametric curve for $0 \leq t \leq 10$.

Do not do problem 5.

5a. Find the point(s) on the curve $r = 1 - \cos \theta$ where the tangent line is horizontal.

5b. Find the point(s) on the curve $r = 1 - \cos \theta$ where the tangent line is vertical.

6. Set up an integral that determines the area common to $r = 4\cos \theta$ and $r = 4\sqrt{3}\sin \theta$.

7. Determine the point on the line segment joining $P(3, 6)$ and $Q(8, -4)$ that is $\frac{2}{3}$ of the way from P to Q .

8. Write parametric equations of the line through the point $(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

9.

$$\mathbf{r}(t) = \left\langle t^2, -\frac{1}{t+1}, 4-t^2 \right\rangle$$

Write parametric equations of the tangent line at the point $(4, 1, 0)$.

10. A ball rolls off a table 4 feet high while moving at a constant speed of 5 feet per second.

10a. How long does it take the ball to hit the floor after it leaves the table?

10b. What is the velocity of the ball as it hits the floor?