

#1

$$\lim_{x \rightarrow \infty} \frac{x}{\ln(1+2e^x)}$$

 $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{1+2e^x} \cdot 2e^x}$$

$$= \lim_{x \rightarrow \infty} \frac{1+2e^x}{2e^x}$$

$$= \lim_{x \rightarrow \infty} \left(\frac{1}{2e^x} + 1 \right) = 1$$

#2

$$\lim_{x \rightarrow \infty} x^3 e^{-x^2}$$

 $\infty \cdot 0$

$$= \lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}}$$

 $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{3x^2}{e^{x^2} \cdot 2x}$$

$$= \lim_{x \rightarrow \infty} \frac{3}{2} \frac{x}{e^{x^2}}$$

 $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{3}{2} \frac{1}{e^{2x} \cdot 2x} = 0$$

#3

$$y = x e^x$$

$$\ln y = \ln x e^x = e^x \ln x$$

$$\frac{1}{y} y' = e^x \frac{1}{x} + e^x \ln x$$

$$y' = y \left(\frac{e^x}{x} + e^x \ln x \right)$$

$$= x e^x \left(\frac{e^x}{x} + e^x \ln x \right)$$

#4

$$y = e^{\cos x} + \cos(e^x)$$

$$y' = e^{\cos x} (-\sin x) - \sin(e^x) e^x$$

$$\#5 \quad \int \sqrt{x} \cos(1+x^{3/2}) dx$$

$$\text{Set } u = 1+x^{3/2}$$

$$du = \frac{3}{2} x^{1/2} dx$$

$$\frac{2}{3} du = \sqrt{x} dx$$

$$\frac{2}{3} \int \cos u du$$

$$= -\frac{2}{3} \cos u + C = -\frac{2}{3} \cos(1+x^{3/2}) + C$$

#6

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_0^{13} \frac{dx}{(1+2x)^{2/3}}$$

$$\text{Set } u = 1+2x$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$x=13$$

$$x=0$$

$$u=1+2x$$

$$u=27$$

$$u=1$$

$$\frac{1}{2} \int_1^{27} u^{-2/3} du = \frac{1}{2} \left. \frac{u^{1/3}}{1/3} \right|_1^{27}$$

$$= \frac{3}{2} \left. u^{1/3} \right|_1^{27}$$

$$= \frac{3}{2} (27)^{1/3} - \frac{3}{2} (1)^{1/3}$$

$$= \frac{9}{2} - \frac{3}{2} = 3$$

#7

$$\int \frac{e^{2x} \tan x}{1 + e^{2x}} dx$$

$$\text{Let } u = 1 + e^{2x}$$

$$du = 2e^{2x} dx$$

$$\begin{aligned} \int \frac{du}{u} &= \ln|u| + C \\ &= \ln|1 + e^{2x}| + C \end{aligned}$$

#8

$$\int e^x \sqrt{1 + e^x} dx$$

$$\text{Let } u = 1 + e^x$$

$$du = e^x dx$$

$$\begin{aligned} \int \sqrt{u} du &= \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{3} (1 + e^x)^{3/2} + C \end{aligned}$$

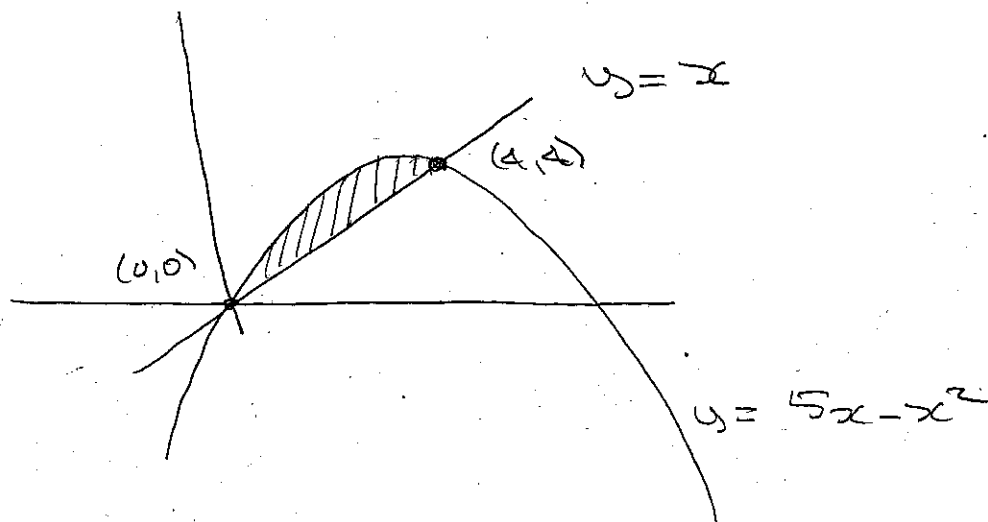
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$$\int \frac{\cos(\ln x)}{x} dx$$

$$\text{Set } u = \ln x \\ du = \frac{1}{x} dx$$

$$\int \cos u du = \sin u + C \\ = \sin(\ln x) + C$$

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points of intersection

$$x = 5x - x^2$$

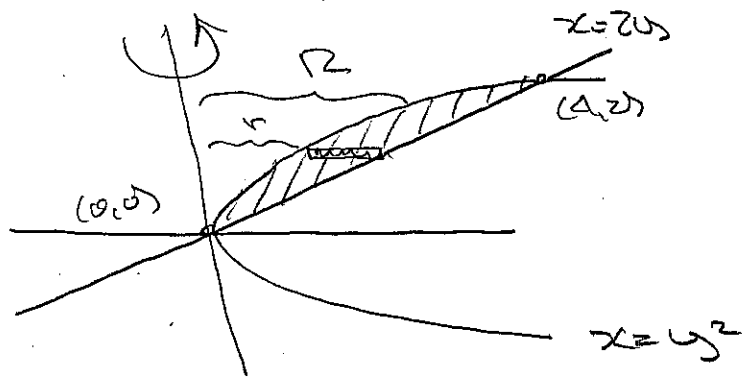
$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x=0, x=4$$

$$\text{Area} = \int_0^4 [(5x - x^2) - x] dx$$

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points of intersection

$$2y = y^2$$

$$y^2 - 2y = 0$$

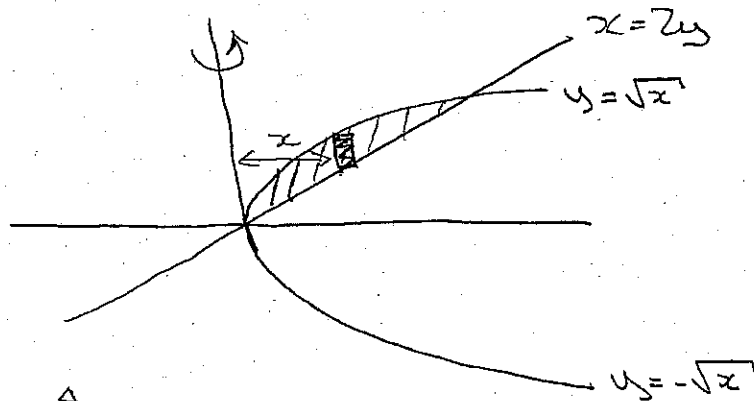
$$y(y - 2) = 0$$

$$y = 0 \quad y = 2$$

washers

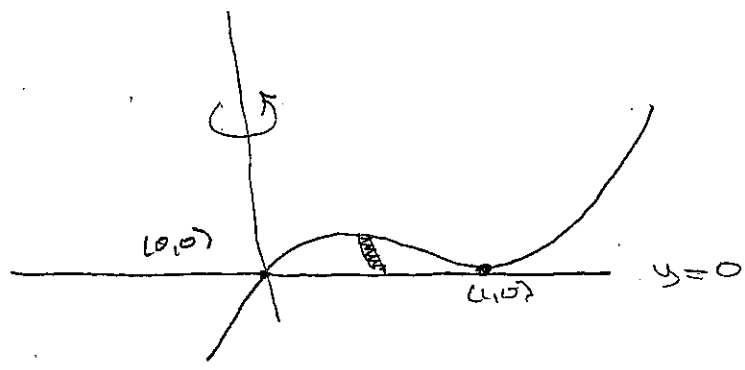
$$\int_0^2 \pi [(2y)^2 - (y^2)^2] dy$$

shells



$$\int_0^1 2\pi x (\sqrt{x} - \frac{1}{2}x) dx$$

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shells

$$\int_0^1 2\pi x [x(x-0^2)] dx$$