#1 $y = x e^{-\frac{1}{2}x}$

$$y' = x e^{-\frac{1}{2}x}\left(\tfrac{1}{2}x^2\right) + e^{-\frac{1}{2}x}$$
$$= \tfrac{1}{2}x e^{-\frac{1}{2}x} + e^{-\frac{1}{2}x}$$

#2 $y = \sqrt{\ln\sqrt{x}}$

$$y = \tfrac{1}{2}(\ln\sqrt{x})^{-\frac{1}{2}} \frac{1}{\sqrt{x}} \tfrac{1}{2}\frac{1}{\sqrt{x}}$$
$$= \tfrac{1}{4}x \frac{1}{\sqrt{\ln\sqrt{x}}}$$

#3 $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$

$$\ln y = 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)$$

$$\frac{1}{y}y' = 4\frac{1}{x^2+1}2x - 3\frac{1}{2x+1}2 - 5\frac{1}{3x-1}3$$

$$y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left[\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right]$$

#4 $\int \frac{\cos(\ln x)}{x}\,dx$.

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \cos u\,du = \sin u + C = \sin(\ln x) + C$

#5 $\int e^{\tan x} \sec^2 x\,dx$.

Let $u = \tan x$

$du = \sec^2 x\,dx$

$\int e^u\,du = e^u + C = e^{\tan x} + C$

#6 $\int \frac{x+1}{x^2+2x}\,dx$

Let $u = x^2+2x$

$du = (2x+2)\,dx$

$\frac{1}{2}\,du = (x+1)\,dx$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+2x| + C$

#7 $\lim_{x\to 0^+} x^2 \ln x$

$0 * -\infty$

$= \lim_{x\to 0^+} \frac{\ln x}{1/x^2}$

$= \lim_{x\to 0^+} \frac{1/x}{-2/x^3} = \lim_{x\to 0^+} \frac{x^3}{-2x}$

$= \lim_{x\to 0^+} \frac{x^2}{-2} = 0$

#8 $\lim_{x\to 0} (\cos 3x)^{5/x}$ 1^∞

Let $y = (\cos 3x)^{5/x}$

$\ln y = \frac{5}{x} \ln(\cos 3x)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{5}{x} \ln(\cos 3x)$

$\infty * 0$

$= \lim_{x\to 0} \frac{5 \ln(\cos 3x)}{x}$

$= \lim_{x\to 0} \frac{5 \frac{1}{\cos 3x}(-\sin 3x)3}{1}$

$= 0$

$\lim_{x\to 0} y = e^0 = 1$