#1. $\int \frac{x^3}{(x^4+1)^4}\,dx$

Let $u = x^4+1$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int \frac{du}{u^4} = \frac{1}{4}\int u^{-4}\,du = \frac{1}{4}\frac{u^{-3}}{-3} + C$

$= -\frac{1}{12}\frac{1}{u^3} + C = -\frac{1}{12}\frac{1}{(x^4+1)^3} + C$

#2 $\int \sin^2 x \cos x\,dx$

Let $u = \sin x$

$du = \cos x\,dx$

$\int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$

#3 $\int_1^2 (x+1)(x^2+2x)^3\,dx$

Let $u = x^2+2x$ $\qquad\qquad$ $x = x^2+2x$

$du = (2x+2)\,dx \qquad x=2 \qquad u=8$

$\frac{1}{2}\,du = (x+1)\,dx \qquad x=1 \qquad u=3$

$$\frac{1}{2}\int_3^8 u^3\,du = \frac{1}{2}\,\frac{u^4}{4}\Big|_3^8 = \frac{1}{8}(8^4-3^4)$$

#4 $\int_{-1}^2 \sqrt{5x+6}\,dx$

Let $u = 5x+6$ $\qquad\qquad$ $u=5x+6$

$du = 5\,dx \qquad x=2 \qquad u=16$

$\frac{1}{5}\,du = dx \qquad x=-1 \qquad u=1$

$$\frac{1}{5}\int_1^{16}\sqrt{u}\,du = \frac{1}{5}\int_1^{16} u^{1/2}\,du = \frac{1}{5}\,\frac{2}{3}\,u^{3/2}\Big|_1^{16}$$

$$= \frac{2}{15}\left(16^{3/2}-1^{3/2}\right) = \frac{2}{15}(64-1)$$

#5

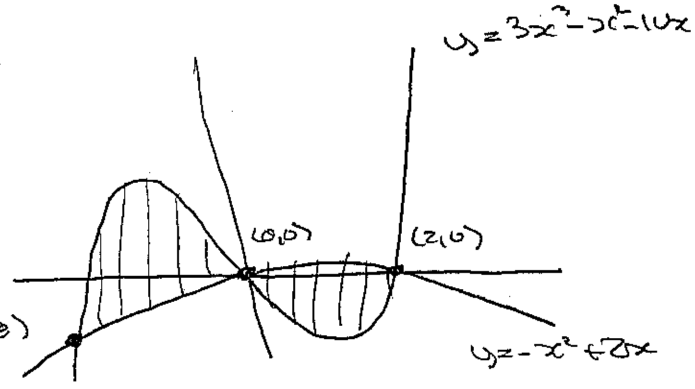
$y = 3x^3 - x^2 - 10x$

$y = -x^2 + 2x$

<u>graph</u>

$-3, 4, 1$

$-10, 10, 1$



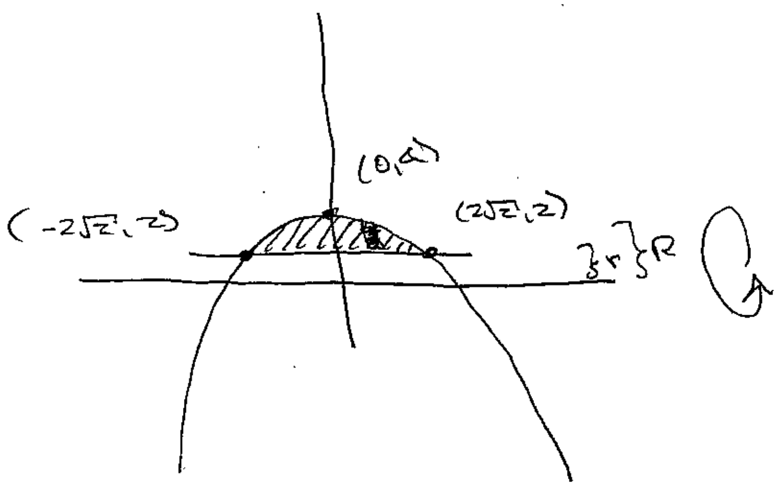
<u>intersections</u>

$$3x^3 - x^2 - 10x = -x^2 + 2x$$
$$3x^3 - 12x = 0$$
$$3x(x^2 - 4) = 0$$
$$x = 0, \quad x = -2, \quad x = 2$$

$$\int_{-2}^{0} \left[(3x^3 - x^2 - 10x) - (-x^2 + 2x)\right] dx$$

$$+ \int_{0}^{2} \left[(-x^2 + 2x) - (3x^3 - x^2 - 10x)\right] dx$$

#6



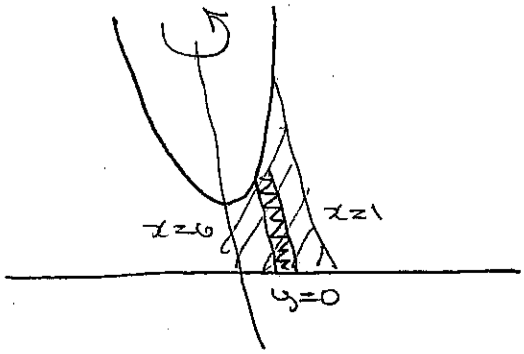
intersections

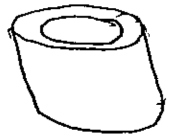
$$4 - \frac{x^2}{4} = y = 2$$
$$16 - x^2 = 8$$
$$x^2 = 8$$
$$x = \pm 2\sqrt{2}$$

washers

$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi \left[\left(4 - \frac{x^2}{4}\right)^2 - 2^2 \right] dx$$

#7





shells

$$\int 2\pi x\, y\, dx$$

$$\int_0^1 2\pi x\,(x^2+1)\,dx$$