Some stuff

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin 2x = 2\sin x \cos x$$

$$\int \sec u \, du = \ln\left|\sec u + \tan u\right| + c$$

$$\frac{1}{1-u} = 1 + u + u^2 + u^3 + u^4 + u^5 + \ldots = \sum_{n=0}^{\infty} u^n \qquad -1 < u < 1$$

$$e^u = 1 + u + \frac{u^2}{2} + \frac{u^3}{3!} + \frac{u^4}{4!} + \frac{u^5}{5!} + \ldots = \sum_{n=0}^{\infty} \frac{u^n}{n!} \qquad -\infty < u < \infty$$

$$\sin u = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \frac{u^9}{9!} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{(2n+1)!} \qquad -\infty < u < \infty$$

$$\cos u = 1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!} + \frac{u^8}{8!} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n}}{(2n)!} \qquad -\infty < u < \infty$$

$$\tan^{-1} u = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \frac{u^9}{9} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} \qquad -1 \le u \le 1$$

$$\ln(1-u) = -u - \frac{u^2}{2} - \frac{u^3}{3} - \frac{u^4}{4} - \ldots = -\sum_{n=1}^{\infty} \frac{u^n}{n} \qquad -1 < u < 1$$

MAT 222 - 001
Fall 2007 Comprehensive Exam
Show all work.

*1. Integrate $\int_2^{\infty} \frac{1}{(x+3)^4}\,dx$. Show all steps.

Determine whether each of the following series (# 2 - 11) is **absolutely convergent**, **conditionally convergent,** or **diverges**. If the series is a convergent geometric series, find its sum. State the test you are using and show all details of the test. Always verify the conditions needed for the test to be valid.

2. $\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+\frac{32}{243}+\cdots$.

3. $\sum_{k=1}^{\infty}\left(\frac{3}{4k}\right)^k$.

4. $\sum_{k=2}^{\infty}\frac{k}{\sqrt{k^5+5}}$.

5. $\sum_{k=1}^{\infty}\frac{1}{(2k+1)\left(\ln(2k+1)\right)^2}$.

6. $\sum_{k=1}^{\infty}\frac{1}{k\sqrt{k+\ln k}}$.

7. $\sum_{k=1}^{\infty} \frac{k^4}{k!}$.

8. $\sum_{k=2}^{\infty} \frac{k^2+1}{k^{3.5}-2}$.

*9. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{1.1}\ln(k+1)}$.

10. $-\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\frac{4}{5}-\cdots$.

11. $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k^3+1}$.

*12. Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n (x+3)^n}{3n}.$$

13. Find the Maclaurin series for $\frac{1}{2+x^2}$. Where does the series converge?

14. Find the function with Maclaurin series $1+x^3+\frac{x^6}{2!}+\frac{x^9}{3!}+\frac{x^{12}}{4!}+\cdots$

*15. Use a Maclaurin series to calculate $\int_0^1 \sin\left(x^2\right) dx$ so that the absolute value of the error is less than 0.00005.