

#1 Let x = number of tickets sold to students
 y = number of tickets sold to faculty
 z = number of tickets sold to general public

$$3x + 5y + 8z = 2572$$

$$x = 3y$$

$$z = 2x$$

*#2

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ -2 & 2 & 1 & 27 \\ 1 & 2 & -3 & 8 \end{array} \right] \begin{array}{l} R_3 - R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array} \right]$$

$$-\frac{1}{3} * R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 0 & -31 & -31 \end{array} \right] \begin{array}{l} R_1 * R_3 \\ R_2 - R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\frac{1}{2} * R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

#3a

$$\begin{aligned}x &= 2 \\ y &= -1 \\ z + u &= 2\end{aligned}$$

#3b

infinitely many solutions

#4

$$\begin{aligned}2x + 10y + s &= 80 \\ 6x + 2y + t &= 72 \\ 3x + 2y - u &= 6 \\ -20x - 30y + v &= 0\end{aligned}$$

$$\begin{array}{c} s \\ t \\ u \end{array} \left[\begin{array}{ccccccc} x & y & z & & & & \\ 2 & 10 & 1 & 0 & 0 & 0 & 80 \\ 6 & 2 & 0 & 1 & 0 & 0 & 72 \\ 3 & 2 & 0 & 0 & -1 & 0 & 6 \\ -20 & -30 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

Phase II

#5a

$$\begin{array}{c} s \\ t \\ u \end{array} \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} 3 & 1 & 2 & 1 & 0 & 0 & 0 & 9 \\ 2 & 3 & 1 & 0 & 1 & 0 & 0 & 8 \\ 1 & 2 & 3 & 0 & 0 & 1 & 0 & 7 \\ -20 & -12 & -18 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} 9/3 \leftarrow \\ 8/2 \\ 7/1 \end{array}$$

least positive

#5b

$$\begin{array}{c} x \\ t \\ u \end{array} \begin{array}{c} y \\ z \end{array} \begin{bmatrix} 3 & 1 & 2 & 1 & 0 & 0 & 0 & 9 \\ 0 & 7 & -1 & -2 & 3 & 0 & 0 & 6 \\ 0 & 5 & 7 & -1 & 0 & 3 & 0 & 12 \\ 0 & -16 & -14 & 20 & 0 & 0 & 3 & 180 \end{bmatrix}$$

Phase I

#6a

$$\begin{array}{c} * s \\ * t \\ * u \end{array} \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 & 0 & 150 \\ 2 & 1 & 0 & 0 & -1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 50 \\ 2 & 2 & 3 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} 150/1 \\ 50/2 \leftarrow \\ 50/0 \end{array}$$

for example ...

#6b

$$\begin{array}{c} s \\ x \\ u \end{array} \begin{array}{c} y \\ z \\ t \end{array} \begin{bmatrix} 0 & 1 & 2 & -2 & 1 & 0 & 0 & 150 \\ 2 & 1 & 0 & 0 & -1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 50 \\ 0 & 1 & 3 & 0 & 1 & 0 & 1 & -50 \end{bmatrix}$$

#7

$$x = 30/6 = 5$$

$$y = 0$$

$$z = 20/2 = 45$$

$$P = 600/3 = 200$$

* #8a

Maximize $P = 9x + 5y$

subject to

$$400x + 200y \leq 2000$$

$$12x + 8.5y \leq 65$$

$$x \geq 0$$

$$y \geq 0$$

* #8b

$$400x + 200y + s = 2000$$

$$12x + 8.5y + t = 65$$

$$-9x - 5y + P = 0$$

	x	y				
s	400	200	1	0	0	2000
t	12	8.5	0	1	0	65
	-9	-5	0	0	1	0

↑

* #8c

#9

$$\begin{bmatrix} 3 & -2 & 7 \\ -2 & 1 & 0 \\ 6 & -5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$$

#10

$$\begin{array}{ccc|cc} & A & & H & & \\ \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -2 & 3 \end{bmatrix} & & & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & & \end{array}$$

$$\begin{bmatrix} 2 & 1 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 2 & 0 \\ 0 & -3 & 5 & | & 1 & 0 & 2 \end{bmatrix}$$

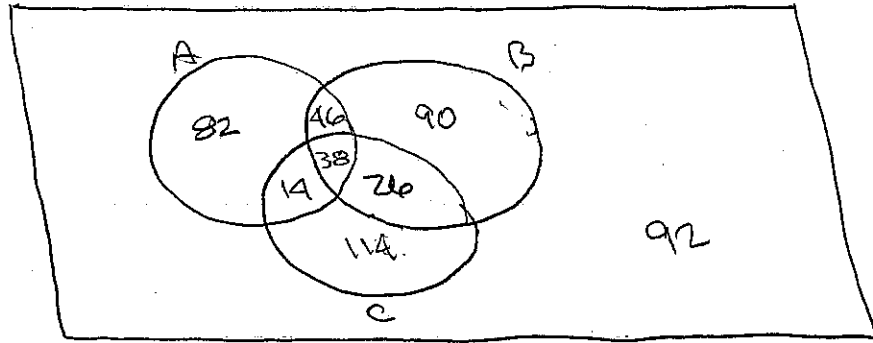
$$\begin{bmatrix} 2 & 0 & 0 & | & 2 & -2 & 0 \\ 0 & 1 & -1 & | & -1 & 2 & 0 \\ 0 & 0 & 2 & | & -2 & 6 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & | & 2 & -2 & 0 \\ 0 & 1 & 0 & | & -1 & 2 & 0 \\ 0 & 0 & 1 & | & -1 & 3 & 1 \end{bmatrix}$$

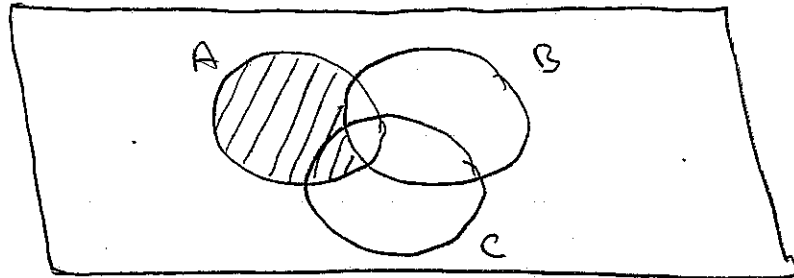
$$\begin{bmatrix} 2 & 0 & 0 & | & 2 & -2 & 0 \\ 0 & 1 & 0 & | & -2 & 5 & -1 \\ 0 & 0 & 1 & | & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & -1 & 0 \\ 0 & 1 & 0 & | & -2 & 5 & -1 \\ 0 & 0 & 1 & | & 1 & 3 & 0 \end{bmatrix}$$

* #11



$A \cap B'$



$$82 + 14 = 96$$

* #12

$A \cap (B \cap C)'$

#13

$$26 * 25 * 24 * 23 * 22 * 21 = P(26, 6) \\ = 165765600$$

#14

$$C(100, 3) = 161700$$

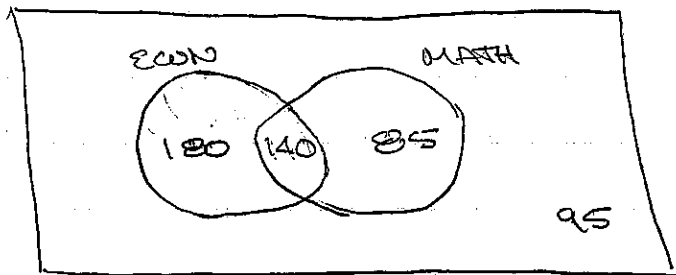
#15

$$C(10, 2) C(10, 3) \\ = 45 * 120 \\ = 5400$$

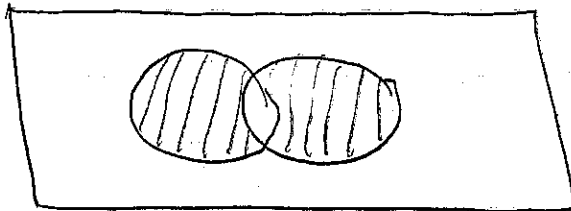
#16

$$7 * 6 * 5 * 4 = P(7, 4) \\ = 840$$

#17



exactly one



$$\frac{180 + 85}{500} = \frac{265}{500} = .53$$

#18

one defective or two defectives

$$\frac{C(20,1)C(4,0) + C(4,2)}{C(24,2)}$$
$$= \frac{20 * 1 + 6}{276} \approx .31159$$

#19

$$P(\text{English}) = \frac{49}{200}$$

$$P(\text{Mathematics}) = \frac{50}{200}$$

$$P(\text{Eng} \cap \text{Math}) = \frac{12}{200}$$

$$P(\text{English} | \text{Mathematics})$$

$$= \frac{P(\text{Eng} \cap \text{Math})}{P(\text{Mathematics})}$$

$$= \frac{12/200}{50/200} = \frac{12}{50} = .24$$

#20

$$P(\text{defective}(A)) = .02$$

$$P(A) = .5$$

$$P(\text{defective}(B)) = .05$$

$$P(B) = .3$$

$$P(\text{defective}(C)) = .06$$

$$P(C) = .2$$

$$P(A|\text{defective}) = \frac{P(\text{defective}(A))P(A)}{P(d(A))P(A) + P(d(B))P(B) + P(d(C))P(C)}$$
$$= \frac{.02 * .5}{.02 * .5 + .05 * .3 + .06 * .2}$$